(1) $\vec{x}(t) = \left(\ln\left(t+\sqrt{t^2+1}\right), \frac{3}{5}\sqrt{t^2+1}, -\frac{4}{5}\sqrt{t^2+1}\right)$

a) $\dot{\vec{x}}(t) = \left(\frac{1}{\sqrt{t^2+1}}, \frac{3}{5}\frac{t}{\sqrt{t^2+1}}, -\frac{4}{5}\frac{t}{\sqrt{t^2+1}}\right) \Rightarrow \|\dot{x}(t)\| = \frac{\sqrt{1+\left(\frac{3}{5}t\right)^2+\left(-\frac{4}{5}t\right)^2}}{\sqrt{t^2+1}} = 1$

$\hookrightarrow s(t) = \int_0^t \|\dot{x}(t')\|\, dt' = t$

b) $\ddot{\vec{x}}(t) = \left(\frac{-t}{(t^2+1)^{3/2}}, \frac{3}{5}\frac{1}{(t^2+1)^{3/2}}, -\frac{4}{5}\frac{1}{(t^2+1)^{3/2}}\right) \Rightarrow \|\ddot{\vec{x}}(t)\| = \frac{\sqrt{t^2+\left(\frac{3}{5}\right)^2+\left(-\frac{4}{5}\right)^2}}{(t^2+1)^{3/2}} = \frac{1}{t^2+1}$

c) Poichè $s(t)=t$, le derivate rispetto ad s sono derivat. rispetto a t.

$\kappa(t) \underset{\uparrow}{=} \|\ddot{\vec{x}}(t)\| = \frac{1}{t^2+1}$

usando $\kappa(s) = \left\|\frac{d^2\vec{x}}{ds^2}\right\|$

versore tangente:

$\vec{t} \underset{\uparrow}{=} \frac{d\vec{x}}{dt} = \left(\frac{1}{\sqrt{t^2+1}}, \frac{3}{5}\frac{t}{\sqrt{t^2+1}}, -\frac{4}{5}\frac{t}{\sqrt{t^2+1}}\right)$

usando: $\vec{t}(s) = \frac{d\vec{x}}{ds}$

$\vec{n} \underset{\uparrow}{=} \frac{1}{\kappa(t)}\ddot{\vec{x}}(t) = \left(\frac{-t}{\sqrt{t^2+1}}, \frac{3}{5}\frac{1}{\sqrt{t^2+1}}, -\frac{4}{5}\frac{1}{\sqrt{t^2+1}}\right)$

usando $\vec{n}(s) = \frac{1}{\kappa(s)}\frac{d^2\vec{x}}{ds^2}$

d) Poichè $\vec{x}(t)$ verifica $4y+3z=0$, la curva è piana.

Di conseguenza, la torsione è zero.

<u>Inutilmente</u>: $\vec{b} = \vec{t} \wedge \vec{n} = \frac{1}{t^2+1}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & \frac{3}{5}t & -\frac{4}{5}t \\ -t & \frac{3}{5} & -\frac{4}{5} \end{vmatrix} = \frac{\frac{4}{5}(1+t^2)\vec{j} + \frac{3}{5}(1+t^2)\vec{k}}{1+t^2}$

$= \frac{4}{5}\vec{j} + \frac{3}{5}\vec{k}$

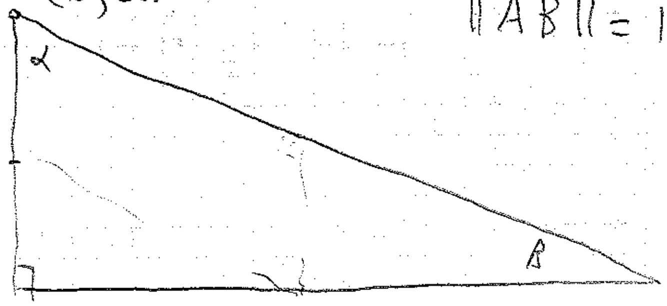
Usando $\frac{d\vec{b}}{ds} = \chi(s)\vec{n}$, risulta $\chi(t) \equiv 0$ (poichè $\vec{b}$ non dipende da t).

Extra: sostituendo $\tau = \ln\left(t+\sqrt{t^2+1}\right)$, otteniamo $\cosh\tau = \sqrt{t^2+1}$ e $\sinh\tau = t$. Nella variabile τ abbiamo

$$\vec{x}(t(\tau)) = \left(\tau, \frac{3}{5}\cosh\tau, -\frac{4}{5}\sinh\tau\right),$$

una catenaria nel piano $4y+3z=0$.

(2) $A(0, \cos\alpha)$ $\quad \|AB\| = 1 \quad \|AC\| = \cos\alpha \quad \|BC\| = \sin\alpha$



$C(0,0)$ $\qquad$ $B(\sin\alpha, 0)$

Baricentri dei lati
↳ infatti, i mediani dei lati

$AB: \left(\frac{1}{2}\sin\alpha, \frac{1}{2}\cos\alpha\right) \to$ massa $= \rho\cdot 1$

$AC: \left(0, \frac{1}{2}\cos\alpha\right) \to$ massa $= \rho\cos\alpha$

$BC: \left(\frac{1}{2}\sin\alpha, 0\right) \to$ massa $= \rho\sin\alpha$

Coordinate del baricentro G:

$$\frac{\rho\left(\frac{1}{2}\sin\alpha, \frac{1}{2}\cos\alpha\right) + \rho\cos\alpha\left(0, \frac{1}{2}\cos\alpha\right) + \rho\sin\alpha\left(\frac{1}{2}\sin\alpha, 0\right)}{1\rho + \rho\cos\alpha + \rho\sin\alpha}$$

$$= \left(\frac{\frac{1}{2}\sin\alpha\left[1+\sin\alpha\right]}{1+\cos\alpha+\sin\alpha}, \frac{\frac{1}{2}\cos\alpha\left[1+\cos\alpha\right]}{1+\cos\alpha+\sin\alpha}\right)$$

Il baricentro G è uguale al baricentro di un sistema di tre punti P_1, P_2, P_3 con masse m_1, m_2, m_3, dove i P_j sono i baricentri dei lati e gli m_j sono le masse dei lati.

(3) $P_1=(1,2,0)$ $\quad P_2=(2,0,-1)$ $\quad P_3=(0,1,2)$

$\vec v_1=-\vec i+\vec j$ $\quad \vec v_2=2\vec i-\vec k$ $\quad \vec v_3=\vec j+2\vec k$

Scegliamo $O=(0,0,0)$.

a) $\vec R=\vec v_1+\vec v_2+\vec v_3=\vec i+2\vec j+\vec k \rightarrow R^2=1^2+2^2+1^2=6$

$$\vec M(O)=\overrightarrow{OP_1}\wedge\vec v_1+\overrightarrow{OP_2}\wedge\vec v_2+\overrightarrow{OP_3}\wedge\vec v_3$$

$$=(\vec i+2\vec j)\wedge\vec v_1+\cancel{\underbrace{(2\vec i-\vec k)}_{=\vec v_2}\wedge\vec v_2}+\cancel{\underbrace{(\vec j+2\vec k)}_{=\vec v_3}\wedge\vec v_3}$$

$$=(\vec i+2\vec j)\wedge(-\vec i+\vec j)=\vec i\wedge\vec j-2\vec j\wedge\vec i=3\vec k$$

$$\vec M(O)\cdot\vec R=3\vec k\cdot(\vec i+2\vec j+\vec k)=3$$

$$\underbrace{\underbrace{\underbrace{\vec k\cdot\vec i}_{=0}\ \ \vec k\cdot\vec j}_{=0}\ \ \vec k\cdot\vec k}_{=3}$$

$$\frac{\vec M(O)\cdot\vec R}{R^2}\vec R=\frac{3}{6}(\vec i+2\vec j+\vec k)=\frac12\vec i+\vec j+\frac12\vec k.$$

b) <u>Metodo I</u>. Sia $\overrightarrow{OA}=\dfrac{\vec R\wedge\vec M(O)}{R^2}=\dfrac{(\vec i+2\vec j+\vec k)\wedge 3\vec k}{6}$

$$=\frac{-3\vec j+6\vec i+\vec 0}{6}=\vec i-\frac12\vec j\longrightarrow A=\left(1,-\frac12,0\right)$$

Asse Centrale: $\{P:\ \overrightarrow{AP}=\lambda\vec R,\ \lambda\in\mathbb{R}\}$

$$(x_P,y_P,z_P)=\left(1,-\frac12,0\right)+\lambda(1,2,1)$$

$$\left.\begin{array}{l}x_P=1+\lambda\\ y_P=-\frac12+2\lambda\\ z_P=\lambda\end{array}\right\}\quad \text{OPPURE:}\quad \left\{\begin{array}{l}x_P=1+z_P\\ y_P=-\frac12+2z_P\end{array}\right.$$

Metodo II: $(x, y, z) \in$ asse centrale se e solo se

$$\frac{x - x_A}{R_x} = \frac{y - y_A}{R_y} = \frac{z - z_A}{R_z}$$

Oppure: $x - 1 = \dfrac{y + \frac{1}{2}}{2} = z$

soluzione alternativa

Prendendo $O' = P_2$, risulta

$$\vec{M}(O') = \overrightarrow{O'P_1} \wedge \vec{v}_1 + \overrightarrow{O'P_3} \wedge \vec{v}_3 = (-\vec{i} + 2\vec{j} + \vec{k}) \wedge (-\vec{i} + \vec{j})$$

$$+ (-2\vec{i} + \vec{j} + 3\vec{k}) \wedge (\vec{j} + 2\vec{k})$$

$$= (\vec{j} + \vec{k}) \wedge (-\vec{i} + \vec{j}) + (-2\vec{i} + \vec{k}) \wedge (\vec{j} + 2\vec{k})$$

$$= (\vec{k} - \vec{j} - \vec{i}) + (-2\vec{k} + 4\vec{j} - \vec{i}) = -2\vec{i} + 3\vec{j} - \vec{k}$$

$$\vec{M}(O') \cdot \vec{R} = (-2\vec{i} + 3\vec{j} - \vec{k}) \cdot (\vec{i} + 2\vec{j} + \vec{k}) = -2 + 6 - 1 = 3$$

$$\frac{\vec{M}(O') \cdot \vec{R}}{R^2} \vec{R} = \frac{1}{2}\vec{i} + \vec{j} + \frac{1}{2}\vec{k}$$

$$\overrightarrow{O'A} = \frac{\vec{R} \wedge \vec{M}(O')}{R^2} = \frac{(\vec{i} + 2\vec{j} + \vec{k}) \wedge (-2\vec{i} + 3\vec{j} - \vec{k})}{6}$$

$$= \frac{1}{6} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ -2 & 3 & -1 \end{vmatrix} = \frac{1}{6}(-5\vec{i} - \vec{j} + 7\vec{k})$$

$$\hookrightarrow A = (2, 0, -1) + \frac{1}{6}(-5, -1, 7) = \left(\frac{7}{6}, -\frac{1}{6}, \frac{1}{6}\right)$$

$$\left.\begin{aligned} x &= \frac{7}{6} + \mu \quad \xrightarrow{\mu = -\frac{1}{6} + \lambda} \quad 1 + \lambda \\ y &= -\frac{1}{6} + 2\mu = -\frac{1}{2} + 2\lambda \\ z &= \frac{1}{6} + \mu = \lambda \end{aligned}\right\}$$

soluzione alternativa: Prendo $O'' = P_3$.

$$\vec M(O'') = \overrightarrow{O''P_1}\wedge \vec v_1 + \overrightarrow{O''P_2}\wedge\vec v_2 = (\vec i+\vec j-2\vec k)\wedge(-\vec i+\vec j)$$
$$+(2\vec i-\vec j-3\vec k)\wedge(\vec j+2\vec k) = (2\vec i - 2\vec k)\wedge(-\vec i+\vec j)$$
$$+(2\vec i-\vec k)\wedge(\vec j+2\vec k) = (2\vec k+2\vec j+2\vec i)+(2\vec k-4\vec j+\vec i)$$
$$=3\vec i-2\vec j+4\vec k.$$

$$\vec M(O'')\cdot\vec R = (3\vec i-2\vec j+4\vec k)\cdot(\vec i+2\vec j+\vec k) = 3-4+4=3$$

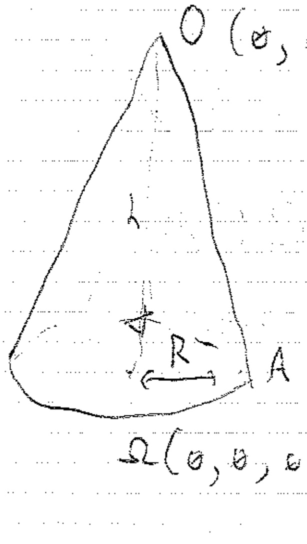
$$\frac{\vec M(O'')\cdot\vec R}{R^2}\vec R = \frac{1}{2}\vec R = \frac{1}{2}\vec i+\vec j+\frac{1}{2}\vec k$$

$$\overrightarrow{O''A} = \frac{\vec R\wedge\vec M(O'')}{R^2} = \frac{(\vec i+2\vec j+\vec k)\wedge(3\vec i-2\vec j+4\vec k)}{6}$$

$$=\frac{1}{6}\begin{vmatrix}\vec i & \vec j & \vec k\\ 1 & 2 & 1\\ 3 & -2 & 4\end{vmatrix} = \frac{5}{3}\vec i-\frac{1}{6}\vec j-\frac{4}{3}\vec k \rightarrow A=\left(\frac{5}{3},\frac{5}{6},\frac{2}{3}\right)$$

$$\left.\begin{aligned} x &= \frac{5}{3}+t \xrightarrow{\lambda = t+\frac{2}{3}} 1+\lambda\\ y &= \frac{5}{6}+2t = -\frac{1}{2}+2\lambda\\ z &= \frac{2}{3}+t = \lambda \end{aligned}\right\}$$

(4)



$$A = (R\cos\theta(t), R\sin\theta(t), 0)$$

$$\dot{\overrightarrow{OA}} = \vec\omega\wedge\overrightarrow{OA}$$

$$\dot\theta R(-\sin\theta,\cos\theta,0) = \vec\omega\wedge(R\cos\theta, R\sin\theta, -h)$$

a) $\vec{v}_A - \underset{\substack{\uparrow \\ =\vec{0}}}{\vec{v}_O} = \vec{\omega} \wedge [(A-\Omega) + \underbrace{(\Omega - O)}_{\text{l'asse di rotazione}}] = \vec{\omega} \wedge (A-O)$, essendo $\vec{\omega} = \underset{\substack{\uparrow \\ \text{di}}}{c}\ \overset{\text{scalare}(t)}{}\ \vec{K}$

Allora

$$\dot{\theta} R(-\sin\theta, \cos\theta, 0) = \vec{\omega} \wedge (R\cos\theta, R\sin\theta, 0)$$

$$= \text{scalare}(t)\ \vec{K} \wedge (R\cos\theta\ \vec{i} + R\sin\theta\ \vec{j})$$

$$= \text{scalare}(t)\ (+R\cos\theta\ \vec{j} - R\sin\theta\ \vec{i})$$

Quindi $\text{scalare}(t) = \dot{\theta}(t) \longrightarrow \boxed{\vec{\omega} = \dot{\theta}\ \vec{K}}$

b) $\vec{v}_A = \dot{\theta} R(-\sin\theta\ \vec{i} + \cos\theta\ \vec{j})$

$$\vec{a}_A = \ddot{\theta} R(-\sin\theta\ \vec{i} + \cos\theta\ \vec{j}) - (\dot{\theta})^2 R(\cos\theta\ \vec{i} + \sin\theta\ \vec{j})$$

$\Longrightarrow \begin{cases} \text{tangenziale}: \vec{a}_{tang.} = \ddot{\theta} R \underset{\substack{\uparrow \\ \text{versore}}}{\vec{t}} \\ \text{centripetale}: \vec{a}_{cent.} = -\dot{\theta}^2 \vec{R}\ \left(= -\dot{\theta}^2 \overrightarrow{\Omega A}\right) \end{cases}$

c) moto $\begin{cases} \text{rotatorio} \\ \to \text{precessione} \begin{cases} \text{asse fisso}: O\Omega \\ \text{asse di precessione}: OA \\ \text{angolo "fisso"}: \angle \Omega O A \\ \quad \left(\text{di tangente } \frac{R}{h}\right) \end{cases} \end{cases}$

$$\boxed{\vec{\omega} = \frac{(A-\Omega) \wedge \vec{v}_A}{h^2}}$$