

Numerical Methods for Inverse Problems

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Many questions in Science and Engineering can be formulated as inverse problems. This kind of problems arise when one is interested in determining the cause of an observed or desired effect. Applications include:

- Helioseismology: Determine the structure of the sun from measurements from earth or space.
- Medical imaging: Electrocardiographic imaging and computerized tomography.
- Image restoration: Determine the unavailable original image from an available contaminated version.

Inverse problems are often ill-posed, i.e., they might not have a unique solution or the solution may be extremely sensitive to errors in the data. Since the data typically is contaminated by measurement or transmission errors, special solution techniques, that yield meaningful results in the presence of errors, have to be applied. For instance, in image restoration problems, the data is an image that has been contaminated by noise and blur.

These lecture will provide an overview of available solution methods. Both methods based on matrix factorization and iterative methods will be discussed. Common factorization methods are based on the singular value decomposition or the generalized singular value decomposition. Factorization methods are well suited for problems of small to medium size. Iterative methods have to be applied when the problem is too large to allow factorization. This holds for many image restoration problems. The unknowns to be determined in these problems are pixel values (in two dimensions) or voxel values (in three dimensions). For instance, 2D images represented by 512×512 pixels have over $2.6 \cdot 10^5$ unknowns. These images do not have very high resolution, but their restoration requires the application of iterative methods. The restoration of high-resolution 2D images and of all 3D images demands the use of iterative methods. We will discuss several popular iterative methods, including conjugate gradient and minimal residual methods, as well as multilevel methods. Both standard methods and recently developed techniques will be discussed. Exercises in MATLAB complement the lectures.