AQuA25

Approximation, Quadrature, and Applications

Santa Margherita di Pula, Italy
October 9–11, 2025

Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani
PIANO NAZIONALE DI RIPRESA E RESILIENZA

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The event is organized within the activity of the *PRIN 2022 PNRR AQuAInt - Approximation and Quadrature for Applicative Integral Models*, based on the collaboration of two research units University of Cagliari and University of Basilicata.

The goal of the conference is to share knowledge and last research on the following topics: approximation by linear and nonlinear operators, numerical integration, orthogonal polynomials, integral equations, inverse problem and applications of integral models, and related issues in numerical linear algebra.

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Abstracts

Keynote Talks

ON NUMERICAL METHODS AND THEIR ANALYSIS FOR DIFFERENT KINDS OF INTEGRAL EQUATIONS

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We restrict ourselves to linear integral equations on a finite interval and to methods for their numerical solution based on global ansatz functions like weighted polynomials or Nyström interpolants. In particular, we consider Fredholm and Volterra integral equations with weakly singular kernel functions as well as strongly singular integral equations with fixed and moving singularities. We discuss the applicability of the Nyström method and its analysis in spaces of continuous functions by using the theory of collectively compact operator sequences. Moreover, we give reasons for the necessity to use other spaces and methods of numerical analysis if we switch from weakly singular to strongly singular integral equations.

AN ARNOLDI-TIKHONOV METHOD FOR THE SOLUTION OF LINEAR ILL-POSED OPERATOR EQUATION

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We are concerned with the solution of linear operator equations with a compact operator. These operators do not have a bounded inverse and therefore the equation has to be regularized before solution. The Arnoldi process provides a convenient way to reduce a compact operator to a nearby operator of finite rank and we regularize with Tikhonov's method. This talk discusses properties of this simple solution approach.

SIMULTANEOUS GAUSSIAN QUADRATURE

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Suppose you want to integrate one function f with respect to $r > 1$ measures $\mu_1, \ldots, \mu_r$ (or weights $w_1, \ldots, w_r$). The goal is to use N function evaluations and to maximize the degree of exactness. This notion of simultaneous quadrature was introduced by Carlos Borges [1] in 1994. It turns out that the optimal choice is to use the zeros of (type II) multiple orthogonal polynomials as quadrature nodes. These quadrature nodes can be computed as the eigenvalues of a banded Hessenberg matrix and the quadrature weights can be obtained using the left and right eigenvectors of this Hessenberg matrix [2] [3]. The Hessenberg matrix is not symmetric and this causes numerical problems. We show how these can be reduced by transforming the matrix [4]. We will illustrate this by giving some examples.

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Invited Talks

EFFICIENT QUADRATURE-BASED PRECONDITIONERS FOR THE RIESZ OPERATOR

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In this talk, we present a comparative analysis of quadrature methods for approximating fractional operators, with a focus on error estimates and their effectiveness in preconditioning the Riesz operator. A first approach uses the Gauss-Jacobi quadrature to approximate this operator as a fractional power of a discretized Laplacian [1, 4]. Others, which achieve faster convergence, rely on Gauss-Laguerre and sinc rules [3, 5]. By appropriately selecting the number of quadrature points, both approaches yield accurate preconditioners that require only a few shifted Laplacian inverses. Numerical tests show that the sinc-based preconditioner is more versatile than the one based on the Gauss-Laguerre rule, and both outperform the Gauss-Jacobi approach [2].

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CONTROL METASTATIC TUMOR GROWTH: FROM MODELING TO NUMERICAL RESULTS

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Cancer is the second most common global cause of death. Modeling metastatic tumor growth with treatment is of paramount importance in developing and advancing knowledge about curing cancer.

One possible approach to model metastatic tumor growth, including also the treatment, is via a coupled size-structured partial differential equation (PDE) and a system of ordinary differential equations (ODEs), the first one describing the evolution in time and size of the metastatic density and the second one describing the evolution in time of the sizes of the primary and secondary tumors, respectively.

The coupled PDE-ODE model, used to describe the metastatic tumor growth, can be reformulated in terms of VIE, whose unknowns are biological observables, such as the cumulative number of metastases and the total metastatic mass.

In this talk I will present a metastatic tumor growth model, which considers the control of the disease by assuming different types of treatment, and an efficient numerical method for the resolution of Volterra integral equations of the second type obtained from the model reformulation, [1], [2].

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NONSTANDARD SOLVERS FOR THE SOLUTION OF FRACTIONAL DIFFERENTIAL MODELS WITH APPLICATIONS

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Over the past few decades, time-fractional differential models have garnered significant attention. They offer a more accurate description of a wide array of natural phenomena and processes in the applied sciences than their integer-order counterparts, especially when modeling systems with memory effects. Solving real-life problems modeled by this type of problems requires reliable and computationally efficient numerical schemes. Reliability is assured by stability and the preservation of the problem's key qualitative behaviors. To this end, we propose two classes of methods for time-fractional reaction-advection-diffusion problems, based on the L1 and Grünwald-Letnikov methods in time and on nonstandard finite differences in space. Both schemes are stable and positivity preserving. We show the effectiveness of these schemes by two significant applications: one from chemical engineering and another concerning the transport of charge carriers in disordered semiconductors, such as those found in lithium batteries.

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ADAPTED NUMERICAL SOLUTION OF REACTION-DIFFUSION PDES IN SEVERAL DIMENSIONS

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Sustainability-related applications, such as vegetation patterns in arid regions and degradation processes in solar cells, can be described by reaction-diffusion Partial Differential Equations (PDEs) in several spatial dimensions. These problems often require long-time integration and fine spatial discretizations, which can be especially demanding in tasks such as parameter estimation, where repeated solution of the PDEs within optimization algorithms may be required. Therefore, the derivation of numerical methods capable of providing a stable and accurate solution in short computing times is crucial. To this aim, we introduce new linearly implicit numerical methods [1], combined with suitable splitting [2] and matrix-oriented techniques (D'Autilia, Sgura, Simoncini, CAMWA, 2020). We analyze the properties of the new methods in terms of accuracy and stability. Numerical experiments confirm the theoretical analysis and underline the good performance of the schemes.

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GALERKIN-BASED EBEM FOR 3D ELASTODYNAMIC. REGULARIZATION OF STRONGLY SINGULAR OPERATORS

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To advance the space-time Energetic Boundary Element Method (EBEM) in 3D elastodynamics [1], we study the characteristic singularity of the double-layer operator, which is involved in the direct boundary integral formulation. On the basis of a decomposition of the time-dependent point load traction Green's function, we employ a regularized Boundary Integral Equation (BIE) and we discretize it by means of a Galerkin-type EBEM (introduced for the first time in [2]) with double analytical integration in the time variable. However, one of the main difficulties of this approach is the efficient approximation of remaining weakly singular double space integrals, whose accurate computation is a key issue for the stability of the method. In particular, the integration domains are generally delimited by the wave fronts of the primary and the secondary waves. By analyzing the geometric characteristic of these domains, we develop an ad-hoc quadrature strategy, where the outer integrals are computed efficiently by the Gaussian quadrature (with a small number of points), while the inner integrals are evaluated with respect to polar coordinates and expressed by analytical formulations. The effectiveness of the proposed approach is illustrated via two benchmark problems.

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COLOR RECONSTRUCTION OF DOMUS DE JANAS MONUMENTS BY PHOTOMETRIC STEREO TECHNIQUES

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Artifacts documentation is an important aspect of archaeological studies, not only to preserve these objects but also to learn from them the culture of ancient populations. Hence, the necessity to digitally document and archive these artifacts. This work is complicated for rock art carvings, that can not be moved from their original location. The photometric stereo technique allows to obtain the three-dimensional digital reconstruction of an object starting from a set of pictures taken with different lighting conditions. When the method is applied to some experimental datasets, computational problems may occur. We deal with these issues by introducing numerical indicators of ideality that allow to figure out if a given dataset is reliable and which images should be selected to better reproduce the object. A color reconstruction it is necessary to the aim, in order to obtain an accurate documentation. In addition, color images contain more information of the surface, provinding data which allows to improve the reconstruction. I will present a study,[1], on the 3D colour reconstruction of engravings found in two Domus de Janas (ancient tombs) in Sardinia, Italy.

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AN ITERATIVE NUMERICAL METHOD FOR SOLVING VOLTERRA-FREDHOLM INTEGRAL EQUATIONS WITH DELAYS ON THE REAL SEMIAXIS

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We are interested in the numerical solution of Volterra-Fredholm integral equations (VFIEs) of the following kind

$$f(t) = \begin{cases} \int_0^t k_1(s,t) f(s-\tau_1) ds + \int_0^\infty k_2(s,t) f(s-\tau_2) e^{-s} ds + g(t), & t \geq 0, \\ f_0(t), & t < 0, \end{cases}$$

where, k_1 and k_2 are given sufficiently smooth kernels, g is the known right-hand side function, f_0 is a known function defining the delay condition, $\tau_i, i = 1, 2,$ represent the delays, and f is the unknown function to be determined.

Many physical phenomena are modeled using delay integral equations. In particular, delay Volterra integral equations are used in engineering for modeling systems with history, such as electric circuits and mechanical systems; delay Volterra-Fredholm integral equations are used in applied sciences for modeling various phenomena such as dynamical systems, physical models and population growth (see, for example, [1]). Due to the presence of delay terms classical methods become less effective or impractical, especially when the problem is defined on unbounded domains.

We propose a Nyström type method, using suitable quadrature rules based on Laguerre zeros, for approximating the solution of the VFIE. An iterative version of the above method is introduced in order to overcome the drawback of loss of accuracy which could occur when long-time solutions of the VFIE have to be computed. Numerical tests will illustrate the effectiveness of the numerical procedure.

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ASYMPTOTIC ESTIMATES OF THE ERROR BOUND FOR GAUSS-RADAU QUADRATURES

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This work deals with the development of an a-priori and derivative-free error analysis for the Gauss-Radau-Jacobi and Gauss-Radau-Laguerre rules for analytic functions. By considering different types of singularities of the integrand functions, we present some strategies that allows to obtain quite accurate approximations of the error, in which the dependence on the number of interior points of the quadrature formula is made explicit. The final error estimates allow to select a-priori the number of quadrature points necessary to achieve a prescribed accuracy.

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APPROXIMATED PRECONDITIONING STRATEGY FOR A NESTED PRIMAL–DUAL SCHEME

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Proximal gradient methods are widely used in imaging, and their convergence can typically be accelerated by incorporating variable metrics and/or extrapolation steps. Recently, it has been shown [1, 2] that preconditioning strategies can further enhance the acceleration of such methods, especially when the proximal operator is computed inexactly via a nested primal-dual solver. However, the computational cost of preconditioning can become significant, particularly for large-scale problems where no exploitable structure is available.

In this talk, we present several approximation strategies for efficiently preconditioning a nested primal-dual method designed to solve regularized convex optimization problems. Numerical experiments in both image deblurring and computed tomography (CT) with total variation regularization confirm that our strategies accelerate convergence while keeping the per-iteration computational cost low.

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NEW SOLUTIONS FOR AN OLD PROBLEM: AN EFFICIENT AND ROBUST SOLVER FOR LINEAR RESPONSE EQUATIONS

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Linear response theory is one of the most powerful tools in computational quantum chemistry, as it allows to directly connect calculation with experimental spectroscopic measurements. In linear response theory, excitation energies and transition moments, that are associated with the positions and intensities of the peaks in spectra, are compute as the eigenvalues and eigenvectors of a generalized eigenvalue problem. As such a problem can be very large, the development of efficient iterative algorithms is paramount. In this contribution, we start from the state of the art algorithm, originally proposed by Olsen almost 40 years ago [1], and using a simple, but effective strategy, recast it into a new iterative procedure which is not only more efficient, but also particularly robust from a numerical point of view [2]. We compare the algorithm not only to the Olsen method, but also to a recent proposed modification of the latter [3], and to a more specialized algorithm that is commonly use in the special case of time-dependent Density Functional Theory [4].

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LAGRANGE INTERPOLATION BASED ON ANTI-GAUSS JACOBI POLYNOMIALS

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In this talk, we present a new interpolation process essentially based on the so-called Anti-Gauss Jacobi nodes, to approximate functions defined in $(-1,1)$ that may have algebraic singularities at ± 1. Laurie [1] first introduced these nodes to construct the so-called anti-Gauss quadrature rules, which have been further used in the numerical treatment of integral equations [2, 3].

Here, we introduce and study a polynomial that interpolates a given function f at these types of nodes. We prove that, under suitable assumptions, the corresponding sequence of Lebesgue constants logarithmically diverges. We also provide estimates of the error in suitable weighted spaces and present numerical tests to support the theoretical investigation.

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STABILITY OF QUADRATURE-BASED METHODS FOR INTEGRAL EQUATIONS WITH DELAY

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This work addresses the numerical solution of Volterra delay integral equations of the form:

$$y(t) = ay(t-\bar{\tau}) + b\int_0^{\bar{\tau}} k(\tau)y(t-\tau)\,d\tau, \quad t \geq 0,$$

where $0 < a < 1$, $b \in \mathbb{R}$, and the kernel function $k : [0, \bar{\tau}] \to \mathbb{R}$ is continuous and positive. The constant $\bar{\tau} > 0$ denotes the fixed delay, and the initial condition is given by a continuous function $\varphi(t)$ defined for $t \in [-\bar{\tau}, 0]$. Such equations, along with their nonlinear generalizations, appear in models for population dynamics, infectious disease transmission, and control systems.We introduce a class of direct quadrature numerical methods for approximating the solution of this equation [1, 2]. The analysis of convergence and stability is carried out. The proposed methods yield approximations whose accuracy is consistent with the order of the truncation error. Furthermore, unconditional stability is established for both Euler and trapezium rules, and constraints on the discretization parameter required for stability are examined in other cases, particularly for the basic test equation.

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ON THE NUMERICAL APPROXIMATION OF FIES VIA DE LA VALLÉE-POUSSIN MEANS

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In this talk, we propose a numerical method for solving the Fredholm Integral Equation (FIE) of the type

$$f(y) - \nu \int_{-1}^{1} f(x)k(x,y)w(x)\,dx = g(y), \quad y \in (-1,1), \tag{1}$$

where $w(x) = (1-x)^{\rho}(1+x)^{\sigma}$, $\rho, \sigma > -1$, is a Jacobi weight, g and k are known functions, ν is a non-zero real parameter and f is the unknown solution.

We focus on the challenging case where the kernel k exhibits pathological behavior. In addition, we allow the right-hand side g to have algebraic singularities at the endpoints. Consequently, we seek the solution of (1) in suitable weighted spaces and provide conditions that guarantee the stability and convergence of the proposed method.

Our approach relies on discrete de la Vallée Poussin means, which are introduced to approximate functions near discontinuities, thus avoiding the typical Gibbs phenomenon and yielding near-best approximations in spaces of continuous functions equipped with a weighted uniform norm [1].

Finally, we present numerical examples that support the theoretical predictions and compare our results with those obtained using numerical methods based on the Lagrange operator [2].

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σ-ORTHOGONAL POLYNOMIALS ON THE SEMICIRCLE

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Orthogonal polynomials on the semicircle and the corresponding quadratures of Gaussian type were introduced and studied by Gautschi and Milovanović for the first time in [1]. In the real case quadrature rules with multiple nodes [3] are observed. A special case of polynomials whose zeros are these multiple nodes are given by the so-called s-orthogonality conditions, and the general case involves σ-orthogonality conditions [2]. The s-orthogonal polynomials on the semicircle are considered in [4]. We introduce σ-orthogonal polynomials on the semicircle and give a detailed study in three special cases where the weight function is the Chebyshev weight function of the first kind.

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MODIFIED TRAPEZOIDAL PRODUCT CUBATURE RULES: DEFINITENESS, MONOTONICITY AND EXIT CRITERIA

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A standard tool for approximation of $I[f] := \int_a^b f(x)\,dx$ is the (n-th) composite trapezium rule $Q_n^{Tr}[f] = h{\sum_{i=0}^{n}}' f(x_i), \quad x_i = a + ih,\ h = \dfrac{b-a}{n}$, where $\sum'$ means that the boundary summands are halved. Assuming the integrand f is convex or concave in $[a,b]$, we have the following well-known properties of the remainder functional $R[Q_n^{Tr};f] := I[f] - Q_n^{Tr}[f]$:

(i) *Definiteness:* $R[Q_n^{Tr};f] \geq 0$ (f convex), $\quad R[Q_n^{Tr};f] \leq 0$ (f concave);

(ii) *Monotonicity:* $\left|R[Q_{2n}^{Tr};f]\right| \leq \dfrac{1}{2}\left|R[Q_n^{Tr};f]\right|$;

(iii) *A posteriori error estimate:* $\left|R[Q_{2n}^{Tr};f]\right| \leq \left|Q_n^{Tr}[f] - Q_{2n}^{Tr}[f]\right|$.

Product trapezium cubature rules are natural candidates for approximating double integrals on a square $[a,b]^2$. For the remainders of appropriately modified trapezium product cubature rules we prove properties analogous to (i)-(iii) in a suitable class of bivariate integrands.

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A REMARK ON A CLASS OF VOLTERRA–FREDHOLM INTEGRAL EQUATIONS ON THE REAL SEMIAXIS

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Volterra–Fredholm integral equations arise in various physical, biological, and financial models, yet the case of unbounded domains has received comparatively little attention in the literature (cf. [1, 2, 3]).

We propose a numerical method based on Lagrange interpolation at Laguerre zeros to approximate the solution of integral equations of the form

$$f(x) - \left[\int_0^{+\infty} h(x,y) w(y) f(y) \mathrm{d}y + \int_0^x k(x-y) w(y) f(y) \mathrm{d}y \right] = g(x) \,,$$

where $x \in (0, +\infty)$, f is the unknown function, $w(x) = x^\alpha e^{-x}$, $\alpha > -1$, and k, h, g are given functions, with $k^{(i)}(0) = 0$ for $i = 0, 1, \ldots, r-1$, $r \in \mathbb{N}$.

We study these equations in suitable weighted spaces of continuous functions and construct a sequence of polynomials converging to the exact solution in weighted uniform norm. We prove the stability and convergence of the method, provide explicit a priori error bounds, and present numerical examples illustrating its effectiveness.

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FUNCTION RECONSTRUCTION FROM INTEGRAL DATA USING EXPONENTIALLY WEIGHTED ENRICHMENTS OF THE CROUZEIX–RAVIART FINITE ELEMENT

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The reconstruction of functions from integral data is a fundamental task in various scientific and engineering applications. In this work, we propose two novel families of weighted polynomial enrichments for the Crouzeix–Raviart finite element to develop accurate approximation operators for bivariate function reconstruction. The proposed enrichments are based on exponential Gegenbauer-weighted quadratic and cubic polynomials, which depend on two parameters, $\sigma > 0$ and $\lambda > -1/2$. These parameters enable better adaptation to localized features of the function being reconstructed, particularly for functions with low regularity or oscillatory behavior. As a result, the proposed enriched finite elements significantly enhance the approximation capabilities of the classical Crouzeix—Raviart finite element, providing greater flexibility and accuracy in capturing sharp gradients, oscillations, and other intricate features. Numerical experiments demonstrate the effectiveness of the proposed approach, showing substantial improvements in reconstruction accuracy compared to the standard Crouzeix–Raviart finite element.

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NON-INVASIVE INVESTIGATION OF SUBSOIL PROPERTIES VIA A TWO-DIMENSIONAL INTEGRAL MODEL

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In this talk, we introduce a two-dimensional first-kind integral model used in applied geophysics to investigate subsoil properties in a non-invasive way. The model describes the interaction between the soil and an electromagnetic device. The aim is to reconstruct the electrical conductivity of the soil from electromagnetic data [1]. The definition of the two-dimensional model is derived, and a numerical study of the forward model based on Gauss-Legendre quadrature formulae is presented. To solve the inverse problem, a linear system obtained from the discretization of the integral equation in the model is considered. The main difficulty is the severe ill-conditioning of the system, so a regularization strategy is applied to obtain good solutions. Some numerical tests show the effectiveness of the proposed approach.

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SOME NEW CLASS OF MULTIPLE ORTHOGONAL POLYNOMIALS

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Multiple orthogonal polynomials are a generalization of orthogonal polynomials in the sense that they satisfy orthogonality conditions with respect to $r \in N$ different weight functions simultaneously. Here, we present multiple orthogonal polynomials that satisfy orthogonality conditions with respect to the set of r bilinear forms defined on the linear space of algebraic polynomials [1], as well as on the linear space of trigonometric polynomials, with special attention to even weight functions. These bilinear forms naturally arise in the construction of sets of anti-Gaussian quadrature rules for the optimal sets of quadrature rules in Borges' sense on the mentioned spaces.

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A NOVEL RATIONAL INTERPOLATION METHOD FOR SOLVING FREDHOLM INTEGRAL EQUATIONS

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In this talk, we present a novel discrete collocation method for the numerical solution of Fredholm integral equations of the second kind in $C[-1,1]$ equipped with the uniform norm. The method is based on a recently introduced rational interpolation scheme developed in the general framework of reproducing kernel Hilbert spaces [3]. This rational approximation has no real poles, uniformly bounded Lebesgue constants and interpolates the target function at arbitrary Jacobi nodes. Moreover, it achieves uniform convergence for all continuous functions with an approximation rate that matches at least that of the best uniform polynomial approximation. These interesting properties are inherited by the numerical method for which stability, convergence and good conditioning are established under minimal assumptions on the integral kernel. A series of numerical experiments confirm the theoretical results and indicate that the proposed method offers a robust and effective alternative in several challenging cases for Nyström-type and polynomial collocation methods based on the same nodes (see, e.g., [1, 2]).

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A CRITICAL VIEW ON THE GLT THEORY: PAST, PRESENT, PERSPECTIVES, AND BEYOND

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The idea of Generalized Locally Toeplitz (GLT) sequences has been introduced as a generalization both of classical Toeplitz sequences and of variable coefficient differential operators and, for every sequence of the class, it has been demonstrated that it is possible to give a rigorous description of the asymptotic spectrum in terms of a function (the symbol) that can be easily identified.

For every $r, d \geq 1$ the r-block d-level GLT class has nice $*$-algebra features and indeed it has been proven that it is stable under linear combinations, products, and inversion when the sequence which is inverted shows a sparsely vanishing symbol (sparsely vanishing symbol $=$ a symbol whose minimal singular value vanishes at most in a set of zero Lebesgue measure). Furthermore, the GLT $*$-algebras virtually include any approximation of partial differential equations (PDEs), fractional differential equations (FDEs), integro-differential equations (IDEs) by local methods (Finite Difference, Finite Element, Isogeometric Analysis etc) and, based on this, we demonstrate that our results on GLT sequences can be used in a PDE/FDE/IDE setting in various directions, including preconditioning, multigrid, spectral detection of branches, fast 'matrix-less' computation of eigenvalues, stability issues, asymptotic low-rank structures, and challenges such as the GLT use in tensors, stochastic, machine learning algorithms. We will discuss also the impact and the further potential of the theory with special attention to new tools and to new directions as those based on symmetrization tricks, on the extra-dimensional approach, and on blocking structures/operations.

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ERROR BOUNDS OF POSITIVE INTERPOLATORY QUADRATURE RULES FOR FUNCTIONS ANALYTIC ON ELLIPSES

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There are lots of specific error bounds of the Gaussian quadrature rules with simple and multiple nodes for functions analytic in a region of the complex plane that contains the interval of integration. They depend on the kind of a quadrature and the measure relative to which the quadrature is considered. We are aware of only one kind of error bound for the standard Gauss quadrature rule with respect to a general measure, given by von Sydow [1], and its generalization to the Gauss-Turán quadrature rule, given by the author [2]. In this paper we consider that kind of the general error bound for the positive interpolatory quadrature rules, in particular for some of their important subclasses. In many numerical experiments we performed (cf. [3]), the results show that the proposed general error bound is of the same range as the existing specific error bounds.

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QUADRATURE RULES RELATED TO ORTHOGONALITY ON THE SEMICIRCLE

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Let D_+ be defined as $D_+ = \{z \in \mathbb{C} : |z| < 1, \mathrm{Im} z > 0\}$ and let Γ be a unit semicircle $\Gamma = \{z = \mathrm{e}^{\mathrm{i}\theta} : 0 \leq \theta \leq \pi\} = \partial D_+$. Let $w(z)$ be a weight function which is positive and integrable on the open interval $(-1,1)$, though possibly singularity at the endpoints, and which can be extended to a function $w(z)$ holomorphic in the half disc D_+. Orthogonal polynomials on the semicircle with respect to the complex-valued inner product

$$(f,g) = \int_\Gamma f(z)g(z)w(z)(\mathrm{i}z)^{-1}\,\mathrm{d}z = \int_0^\pi f(\mathrm{e}^{\mathrm{i}\theta})g(\mathrm{e}^{\mathrm{i}\theta})w(\mathrm{e}^{\mathrm{i}\theta})\,\mathrm{d}\theta$$

was introduced by Gautschi and Milovanović in [2] (for $w(x) = 1$), where the certain basic properties were proved. Such orthogonality as well as the applications involving Gauss-Christoffel quadrature rules were further studied in [1] and [5]. Inspired by Laurie's paper [3], Milosavljević at el. in [4] introduced anti-Gaussian quadrature rules related to the orthogonality on the semicircle, presented some of their properties, and suggested a stable numerical method for their construction. In this lecture we introduce the generalized averaged Gaussian quadrature rules on the semicircle. Two methods for their construction and some properties are included. In addition, the accuracy of such quadrature rules and applications are demonstrated through numerical examples.

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QUADRATURE RULES WITH QUASI-DEGREE OF EXACTNESS

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Many quadrature rules are designed to be exact for easily integrable functions similar to the integrand. The Gauss formula with n nodes is exact on the space of all polynomials of degree $\leq 2n-1$ and it represents a unique optimal interpolatory quadrature rule. It is suitable for application if the integrand is polynomial-like. Note that a function can be similar to certain polynomials, but not similar to some other polynomials. This motivates us to construct a quadrature rule exact only on a subspace of polynomials that share certain properties with the integrand. After choosing m arbitrary points x_k (at which the integrand is defined), we transform the given integral into a sum of an integral that does not cause a quadrature error and an integral with a property that the points x_k are the zeros of its modified integrand. Then, we approximate the integral of the modified integrand by an n-point formula exact on the subspace of polynomials of degree $\leq 2n-1+m$ with m fixed zeros x_k (those fixed zeros are not a disadvantage, since the modified integrand has the same zeros) – such a formula is said to have a quasi-degree of exactness $2n-1+m$.

AN APPROACH FOR HIGHER ORDER APPROXIMATION BY MODIFIED GOODMAN-SHARMA VARIANTS OF CERTAIN CLASSICAL OPERATORS

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We present new modifications of Goodman-Sharma type variants of the classical Bernstein/Baskakov/Meyer-König and Zeller operators for approximation of continuous functions on $[0,1]$, $[0,\infty)$, $[0,1)$, respectively.

Our approach gives new operators which are linear but not positive. In each case we prove a direct inequality and a strong converse inequality with estimates in the sense of a related K-functional. Moreover, the suggested operators have the advantage of second order rate of approximation, compared with the first order for the classical operators, see [1, 2, 3].

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A COMPARISON OF MATLAB CODES FOR SOLVING FRACTIONAL DIFFERENTIAL EQUATIONS OF CAPUTO TYPE

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Fractional differential equations have, over the previous decades, attracted more and more attention from the mathematical community, due to both their interesting analytical properties and widespread applicability in scientific modeling. While there are multiple, typically non-equivalent definitions of fractional derivatives in use, equations with Caputo-type fractional operators are perhaps most often used, since they allow, among other things, the use of "more natural" initial conditions for problems of fractional order greater than one. As a result, many competing numerical methods have been devised and investigated for approximating their solution; for some such methods, their Matlab codes have also been made available. The aim of the present contribution is to is to highlight a systematic comparison of some of such codes on a selected set of test problems, published in [1], and to introduce the FDE-testset environment available on the website https://people.dimai.unifi.it/brugnano/FDEtestset.

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Posters

THE WEAK ORDER FOR STRATONOVICH STOCHASTIC DIFFERENTIAL EQUATION USING THE TRIVIAL METHOD

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This paper investigates the use of weak convergence in Stratonovich stochastic differential equations (SDEs), shifting the focus from the robust convergence techniques previously employed. We introduce a novel application of the trivial coupling method within the weak convergence framework, specifically addressing non-invertible equations. Our approach simplifies the handling of random scenarios and computational tasks, with potential applications spanning physics, biology, and engineering. We provide a detailed account of the method, including its theoretical background and practical implementation using MATLAB. Our results confirm the validity of our approach, demonstrating its effectiveness even with degenerate diffusion coefficients. This advancement in weak convergence strategies offers new insights and practical solutions for complex systems and opens avenues for further research.

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LOW RANK AND SPARSE SPLITTING FOR GLYPH EXTRACTION

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This work is concerned about the extraction of archaeological glyphs from a surface by extending the approach introduced in a previous work [1]. As in that work, the input surface is decomposed into the sum of two components: a smooth surface representing the underlying general shape, and a sparse one capturing the fine details, including the glyphs. However, instead of relying on the Fractional Laplacian differential operator to extract the smooth component, we propose a novel variational framework that promotes low-rank matrices via nuclear norm minimization. This formulation leads to a convex optimization problem that is less sensitive to boundary conditions and effectively isolates the localized features associated with the glyphs. In addition, we explore non-convex sparsity models by incorporating quasi-norms ℓ_q with $q < 1$, which further enhance the separation of fine structures. We evaluate the effectiveness of the method on both synthetic and real archaeological datasets and propose strategies for reducing the computational cost of the algorithm.

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ITERATED $\ell^2 - \ell^q$ REGULARIZATION

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In many areas of science and engineering we are faced by linear inverse problems. The solution of this kind of problems is very sensitive to perturbations in the data. To reduce the sensitivity of the computed solution to perturbations in the data, one employs regularization. Instead of solving the original problem, one may solve an ℓ^2-ℓ^q minimization problem, i.e., one minimizes a weighted sum of a squared Euclidean norm of a fidelity term and the qth power of the ℓ^q-norm with $0 < q \leq 2$ of a regularization term, where we note that the "ℓ^q-norm" does not satisfy all properties of a norm for $0 < q < 1$, see, e.g., [2, 3] and references within. This poster describes an iterated variant of this regularization approach [1]. It is known that iterated variants of Tikhonov regularization yields computed solution of higher quality than "standard" Tikhonov regularization. We show that iterated ℓ^2-ℓ^q minimization gives computed solutions of higher quality than standard ℓ^2-ℓ^q minimization. Computed examples illustrate the performance of the proposed method.

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A Nyström method for 2D Fredholm integral equations based on anti-Gaussian cubature formulae

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Fredholm integral equations defined on the square are considered and a numerical method of Nyström-type is proposed for their numerical solution. The method is based on Anti-Gauss cubature rules, proposed in [1] for the first time; see also [2] for the numerical implementation. The stability, convergence, and conditioning of the proposed Nyström-type method are studied in suitable weighted spaces. The numerical solution of the resulting dense linear system is also investigated and several numerical tests are presented.

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EXTREMAL FUNCTION AND SEQUENCE FOR HARDY INEQUALITIES IN L_p AND l_p

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The behaviour of the smallest possible constants $d(a,b)$ and d_n in classical Hardy inequalities

$$\int_a^b \left(\frac{1}{x}\int_a^x f(t)dt\right)^p dx \leq d(a,b) \int_a^b f^p(x)dx$$

and

$$\sum_{k=1}^n \left(\frac{1}{k}\sum_{j=1}^k a_j\right)^p \leq d_n \sum_{k=1}^n a_k^p$$

is discussed. For $p = 2$ the exact constant $d(a,b)$ and the exact rate of convergence of d_n are established and the extremal function and the "almost extremal" sequence are found. For $2 < p < \infty$ the exact rate of convergence of $d(a,b)$ and d_n are established and the "almost extremal" function and sequence are found.

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ON THE SPECTRAL DISTRIBUTION AND MAXIMAL RESULTS FOR GEOMETRIC MEANS OF HPD GLT MATRIX SEQUENCES

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In this work, we extend our previous analysis [1] on the spectral distribution of the geometric mean of matrix-sequences formed by Hermitian Positive Definite (HPD) matrices under the framework of Generalized Locally Toeplitz (GLT) $*$-algebra. Building upon our earlier results, we address the necessity of the invertibility assumption on the GLT symbols for ensuring that the geometric mean admits the expected spectral distribution. Motivated by the fact that inversion is required mainly due to non-commutativity, we consider the case where the GLT symbols commute and rigorously prove that the invertibility assumption can be relaxed, thereby resolving a longstanding conjecture in the field.

Furthermore, we extend the spectral analysis to the Karcher mean for more than two HPD GLT matrix-sequences, showing that the resulting mean remains within the GLT class, with the symbol given by the geometric mean of the individual symbols. Numerical experiments validate our theoretical findings and illustrate the extremal spectral behavior and the emergence of GLT momentary symbols in the case of non-commuting, degenerate symbols, where the standard distribution formula fails or becomes ill-posed.

Finally, our results are extended to the multilevel block case (for $r, d \geq 1$), offering a broader generalization and deeper numerical validation for the spectral theory of geometric means of structured matrix-sequences.

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QUADRATURE ERROR ESTIMATES FOR KERNELS WITH LOGARITHMIC SINGULARITY

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In this work, we present accurate error estimates for two commonly used quadrature rules—the Gauss-Legendre and trapezoidal rules—when applied to the numerical evaluation of two-dimensional layer potentials with logarithmic singularities. Accurately estimating quadrature error is crucial in boundary integral methods, especially as the evaluation point nears the boundary of the domain. In such cases, the integrals become nearly singular, and the standard quadrature error can grow rapidly. Our results provide a way to quantify this error growth, helping practitioners to recognize when standard methods no longer provide sufficient accuracy and specialized quadrature techniques are necessary. These estimates represent the fundamental key to build novel efficient numerical solvers for boundary integral equations.

APPROXIMATION OF FRACTIONAL DERIVATIVES IN ZYGMUND-HÖLDER SPACES OF FUNCTIONS

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We present a numerical method for approximating Hadamard finite-part integrals of the type

$$H_{p,\nu}(f,y) = \int_0^y \frac{f(x)}{(y-x)^{p+1+\nu}}dx, \quad p \in \mathbf{N},\ 0 < \nu < 1,\ y \in (0,1),$$

proving that it is stable and convergent in Zygmund-Hölder spaces.

In view of the relations of $H_{p,\nu}(f,y)$ with Riemann-Liouville and Caputo fractional derivatives (e.g. [1, 2])

$$(D_{RL}^{p+\nu} f)(y) = \frac{1}{\Gamma(-p-\nu)} H_{p,\nu}(f,y),$$

$$(D_{C}^{p+\nu} f)(y) = (D_{RL}^{p+\nu} f)(y) - \sum_{k=0}^{p} \frac{(D^k f)(0)}{\Gamma(k-p-\nu+1)} y^{k-p-\nu}.$$

such a new method has been employed to approximate them with high accuracy. Some numerical tests are provided, that confirm the theoretical estimates.

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ASYMPTOTIC BEHAVIOR OF THE MODULUS OF THE KERNEL AND ERROR BOUNDS OF ANTI-GAUSSIAN QUADRATURE FORMULAS WITH JACOBI WEIGHTS

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The remainder term of anti-Gaussian quadrature rules for analytic integrands with respect to Jacobi weight functions $\omega_{a,b}(x) = (1-x)^a(1+x)^b$, where $a, b > -1$, is analyzed, and sharp estimates of the error are provided. These kinds of quadrature formulas were introduced by D.P. Laurie and have been recently studied by M.M. Spalević for the case of Chebyshev-type weight functions ω.

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NYSTRÖM METHODS FOR NONLINEAR VOLTERRA INTEGRAL EQUATIONS

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We consider Nonlinear Volterra Integral Equations (NVIEs) of type

$$y(s) + \mu \int_a^s k(t,s) f(t, y(t)) (s-t)^{\alpha} (t-a)^{\beta} \, dt = g(s), \quad s \in (a,b], \tag{2}$$

where $\alpha, \beta > -1$, k,g and f are given functions, and y is the unknown. While equations of type (2) appear in numerous applications, this study specifically examines NVIEs as a reformulation (see, for example [1]) of equivalent nonlinear Fractional Differential Equations (FDEs). Here we introduce two Nyström-type methods based on product-type polynomial quadrature formulae. In particular, following an idea proposed in [2], the first formula is based on the polynomial approximation using the Generalized Bernstein polynomials, whereas, inspired by [3], the second is constructed using the Lagrange polynomials. The resulting Nyström methods are stable and convergent. Some examples of both NVIE and FDE numerical resolution are also given.

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SYMBOL-BASED MULTILEVEL BLOCK τ PRECONDITIONERS FOR MULTILEVEL BLOCK TOEPLITZ SYSTEMS: GLT-BASED ANALYSIS AND APPLICATIONS

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Preconditioning techniques for Toeplitz linear systems have been a prominent area of research for several decades, particularly in the real symmetric (or, more generally, Hermitian) case, where many effective strategies have been developed. In contrast, the real nonsymmetric case remains less explored, mainly due to the challenges associated with analyzing the eigenvalues and, consequently, the convergence behavior of iterative solvers.

To address this issue, we employ a symmetrization technique that transforms the coefficient matrix into a real symmetric Hankel structure with a known eigenvalue distribution. By leveraging Generalized Locally Toeplitz (GLT) theory, we then develop a novel preconditioning strategy involving centrosymmetric preconditioners, such as those derived from the τ algebra. This approach constitutes a general framework, as it relies solely on the generating function of the Toeplitz matrix, under the mild assumption that it is well-defined. Moreover, the results can be extended to both the multilevel and the block setting.

Finally, we demonstrate the effectiveness of this approach by applying it to large, dense and ill-conditioned linear systems arising from the discretization of space-fractional diffusion equations. Through a series of numerical experiments, we assess the performance of our proposal against several state-of-the-art preconditioners.

A NEW CLASS OF QUADRATURE RULES FOR ESTIMATING THE ERROR IN GAUSS QUADRATURE

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Averaged Gaussian quadrature rules are introduced as alternatives to the Gauss-Kronrod quadrature rules, when we use them for estimating the error of the corresponding Gauss quadrature rules. Their lack might be that they are not always internal. In those cases we introduce the new averaged Gaussian quadratures NAG, which can be used as an alternative to the averaged Gaussian quadrature rules, especially in the cases when the former rules are internal and the latter are not. We present here in short a part of the results that are obtained jointly with A. V. Pejčev, L. Reichel and M. M. Spalević (cf. [1]).

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SPACE–NONLOCAL DESCRIPTION OF CELL MOVEMENT

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We propose a new space–nonlocal model based only on integral terms describing cell movement caused by various interactions. These terms retain the basic properties of the corresponding space–local (that is, differential) terms. Interactions between real objects take place at a certain distance between them, which may be caused, for example, by their geometric shapes. Unlike the local (differential) model, the proposed space–nonlocal model does not exhibit blow–ups of solutions. On the other hand, the nonlocal terms approximate the corresponding local terms. The model refers to cancer invasion on surrounding tissue, but the generalization to any movement of biological object is straightforward. The performed numerical simulations show how nonlocal effects affect the dynamics of the system under consideration.

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