AN ITERATIVE NUMERICAL METHOD FOR SOLVING VOLTERRA-FREDHOLM INTEGRAL EQUATIONS WITH DELAYS ON THE REAL SEMIAXIS

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We are interested in the numerical solution of Volterra-Fredholm integral equations (VFIEs) of the following kind

$$f(t) = \begin{cases} \int_0^t k_1(s,t) f(s-\tau_1) ds + \int_0^\infty k_2(s,t) f(s-\tau_2) e^{-s} ds + g(t), & t \geq 0, \\ f_0(t), & t < 0, \end{cases}$$

where, k_1 and k_2 are given sufficiently smooth kernels, g is the known right-hand side function, f_0 is a known function defining the delay condition, $\tau_i, i = 1, 2,$ represent the delays, and f is the unknown function to be determined.

Many physical phenomena are modeled using delay integral equations. In particular, delay Volterra integral equations are used in engineering for modeling systems with history, such as electric circuits and mechanical systems; delay Volterra-Fredholm integral equations are used in applied sciences for modeling various phenomena such as dynamical systems, physical models and population growth (see, for example, [1]). Due to the presence of delay terms classical methods become less effective or impractical, especially when the problem is defined on unbounded domains.

We propose a Nyström type method, using suitable quadrature rules based on Laguerre zeros, for approximating the solution of the VFIE. An iterative version of the above method is introduced in order to overcome the drawback of loss of accuracy which could occur when long-time solutions of the VFIE have to be computed. Numerical tests will illustrate the effectiveness of the numerical procedure.

References

[1] Z. Yang, H. Brunner, *Blow-up behavior of Hammerstein-type delay Volterra integral equations*, Front. Math. China, 8 (2), 261-280, 2013

[2] I.M. Bulai, M.C. De Bonis, C. Laurita, *A new MATLAB software for numerical computation of biological observables for metastatic tumor growth*, Mathematics and Computers in Simulation, 234, 31-49, 2025.