EXTREMAL FUNCTION AND SEQUENCE FOR HARDY INEQUALITIES IN L_p AND l_p

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The behaviour of the smallest possible constants $d(a,b)$ and d_n in classical Hardy inequalities

$$\int_a^b \left(\frac{1}{x}\int_a^x f(t)dt\right)^p dx \leq d(a,b) \int_a^b f^p(x)dx$$

and

$$\sum_{k=1}^n \left(\frac{1}{k}\sum_{j=1}^k a_j\right)^p \leq d_n \sum_{k=1}^n a_k^p$$

is discussed. For $p = 2$ the exact constant $d(a,b)$ and the exact rate of convergence of d_n are established and the extremal function and the "almost extremal" sequence are found. For $2 < p < \infty$ the exact rate of convergence of $d(a,b)$ and d_n are established and the "almost extremal" function and sequence are found.

References

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