STABILITY OF QUADRATURE-BASED METHODS FOR INTEGRAL EQUATIONS WITH DELAY

E. Messina, and C. Panico
Dept. of Mathematics and Applications, Univ. of Naples Federico II
via Cintia, I-80126, Naples, Italy
eleonora.messina@unina.it

This work addresses the numerical solution of Volterra delay integral equations of the form:

$$y(t) = ay(t-\bar{\tau}) + b\int_0^{\bar{\tau}} k(\tau)y(t-\tau)\,d\tau, \quad t \geq 0,$$

where $0 < a < 1$, $b \in \mathbb{R}$, and the kernel function $k : [0, \bar{\tau}] \to \mathbb{R}$ is continuous and positive. The constant $\bar{\tau} > 0$ denotes the fixed delay, and the initial condition is given by a continuous function $\varphi(t)$ defined for $t \in [-\bar{\tau}, 0]$. Such equations, along with their nonlinear generalizations, appear in models for population dynamics, infectious disease transmission, and control systems.We introduce a class of direct quadrature numerical methods for approximating the solution of this equation [1, 2]. The analysis of convergence and stability is carried out. The proposed methods yield approximations whose accuracy is consistent with the order of the truncation error. Furthermore, unconditional stability is established for both Euler and trapezium rules, and constraints on the discretization parameter required for stability are examined in other cases, particularly for the basic test equation.

References

[1] C. T. H. Baker, N. J. Ford *Some Applications of the Boundary-Locus Method and the Method of D-Partitions*, IMA Journal of Numerical Analysis 11 (2) pp. 143-158 (1991).

[2] E. Messina, E. Russo, A. Vecchio. *Comparing analytical and numerical solution of a nonlinear two-delay integral equations.* Mathematics and Computers in Simulation 81, pp. 1017–1026 (2011).