ON THE NUMERICAL APPROXIMATION OF FIES VIA DE LA VALLÉE-POUSSIN MEANS

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In this talk, we propose a numerical method for solving the Fredholm Integral Equation (FIE) of the type

$$f(y) - \nu \int_{-1}^{1} f(x)k(x,y)w(x)\,dx = g(y), \quad y \in (-1,1), \tag{1}$$

where $w(x) = (1-x)^{\rho}(1+x)^{\sigma}$, $\rho, \sigma > -1$, is a Jacobi weight, g and k are known functions, ν is a non-zero real parameter and f is the unknown solution.

We focus on the challenging case where the kernel k exhibits pathological behavior. In addition, we allow the right-hand side g to have algebraic singularities at the endpoints. Consequently, we seek the solution of (1) in suitable weighted spaces and provide conditions that guarantee the stability and convergence of the proposed method.

Our approach relies on discrete de la Vallée Poussin means, which are introduced to approximate functions near discontinuities, thus avoiding the typical Gibbs phenomenon and yielding near-best approximations in spaces of continuous functions equipped with a weighted uniform norm [1].

Finally, we present numerical examples that support the theoretical predictions and compare our results with those obtained using numerical methods based on the Lagrange operator [2].

References

[1] W. Themistoclakis, *Uniform approximation on* $[-1,1]$ *via discrete de la Vallée Poussin means*, Numer. Algorithms, 60 (2012), pp. 593–612.

[2] M.C. De Bonis, G. Mastroianni, *Projection methods and condition numbers in uniform norm for Fredholm and Cauchy singular integral equations*, SIAM J. Numer. Anal., 44, 4 (2006), pp. 1351–1374.