Modified trapezoidal product cubature rules: definiteness, monotonicity and exit criteria

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A standard tool for approximation of $I[f] := \int_a^b f(x)\,dx$ is the (n-th) composite trapezium rule $Q_n^{Tr}[f] = h\sum_{i=0}^{n}{}' f(x_i), \quad x_i = a + ih,\ h = \dfrac{b-a}{n}$, where $\sum'$ means that the boundary summands are halved. Assuming the integrand f is convex or concave in $[a,b]$, we have the following well-known properties of the remainder functional $R[Q_n^{Tr};f] := I[f] - Q_n^{Tr}[f]$:

(i) *Definiteness:* $R[Q_n^{Tr};f] \geq 0$ (f convex) , $\quad R[Q_n^{Tr};f] \leq 0$ (f concave);

(ii) *Monotonicity:* $\left|R[Q_{2n}^{Tr};f]\right| \leq \dfrac{1}{2}\left|R[Q_n^{Tr};f]\right|$;

(iii) *A posteriori error estimate:* $\left|R[Q_{2n}^{Tr};f]\right| \leq \left|Q_n^{Tr}[f] - Q_{2n}^{Tr}[f]\right|$.

Product trapezium cubature rules are natural candidates for approximating double integrals on a square $[a,b]^2$. For the remainders of appropriately modified trapezium product cubature rules we prove properties analogous to (i)-(iii) in a suitable class of bivariate integrands.

References

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