A REMARK ON A CLASS OF VOLTERRA–FREDHOLM INTEGRAL EQUATIONS ON THE REAL SEMIAXIS

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Volterra–Fredholm integral equations arise in various physical, biological, and financial models, yet the case of unbounded domains has received comparatively little attention in the literature (cf. [1, 2, 3]).

We propose a numerical method based on Lagrange interpolation at Laguerre zeros to approximate the solution of integral equations of the form

$$f(x) - \left[\int_0^{+\infty} h(x,y)w(y)f(y)\mathrm{d}y + \int_0^{x} k(x-y)w(y)f(y)\mathrm{d}y\right] = g(x)\,,$$

where $x \in (0,+\infty)$, f is the unknown function, $w(x) = x^{\alpha}e^{-x}$, $\alpha > -1$, and k,h,g are given functions, with $k^{(i)}(0) = 0$ for $i = 0,1,\ldots,r-1$, $r \in \mathbb{N}$.

We study these equations in suitable weighted spaces of continuous functions and construct a sequence of polynomials converging to the exact solution in weighted uniform norm. We prove the stability and convergence of the method, provide explicit a priori error bounds, and present numerical examples illustrating its effectiveness.

References

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