APPROXIMATION OF FRACTIONAL DERIVATIVES IN ZYGMUND-HÖLDER SPACES OF FUNCTIONS

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We present a numerical method for approximating Hadamard finite-part integrals of the type

$$H_{p,\nu}(f,y) = \int_0^y \frac{f(x)}{(y-x)^{p+1+\nu}} dx, \quad p \in \mathbf{N},\ 0 < \nu < 1,\ y \in (0,1),$$

proving that it is stable and convergent in Zygmund-Hölder spaces.

In view of the relations of $H_{p,\nu}(f,y)$ with Riemann-Liouville and Caputo fractional derivatives (e.g. [1, 2])

$$(D_{RL}^{p+\nu} f)(y) = \frac{1}{\Gamma(-p-\nu)} H_{p,\nu}(f,y),$$

$$(D_{C}^{p+\nu} f)(y) = (D_{RL}^{p+\nu} f)(y) - \sum_{k=0}^{p} \frac{(D^k f)(0)}{\Gamma(k-p-\nu+1)} y^{k-p-\nu}.$$

such a new method has been employed to approximate them with high accuracy. Some numerical tests are provided, that confirm the theoretical estimates.

References

[1] K. Diethelm, The analysis of Fractional Differential equations, Springer Heidelberg Dordrecht London New York, 2010.

[2] D. Elliott, *Three algorithms for Hadamard finite-part integrals and fractional derivatives*, JCAM (1995), **62**, 267-283.