NYSTRÖM METHODS FOR NONLINEAR VOLTERRA INTEGRAL EQUATIONS

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We consider Nonlinear Volterra Integral Equations (NVIEs) of type

$$y(s) + \mu \int_a^s k(t,s) f(t, y(t)) (s-t)^{\alpha} (t-a)^{\beta} \, dt = g(s), \quad s \in (a,b], \tag{1}$$

where $\alpha, \beta > -1$, k,g and f are given functions, and y is the unknown. While equations of type (1) appear in numerous applications, this study specifically examines NVIEs as a reformulation (see, for example [1]) of equivalent nonlinear Fractional Differential Equations (FDEs). Here we introduce two Nyström-type methods based on product-type polynomial quadrature formulae. In particular, following an idea proposed in [2], the first formula is based on the polynomial approximation using the Generalized Bernstein polynomials, whereas, inspired by [3], the second is constructed using the Lagrange polynomials. The resulting Nyström methods are stable and convergent. Some examples of both NVIE and FDE numerical resolution are also given.

References

[1] A. A. Kilbas, H. M. Srivastava, J. J. Trujillo, *Theory and applications of fractional differential equations*, North-Holland Mathematics Studies, Vol. 204, Elsevier, (2006).

[2] L. Fermo, D. Mezzanotte, D. Occorsio, *On the numerical solution of Volterra integral equations on equispaced nodes*, Electron. Trans. Numer. Anal. 59 (2023), pp. 9–23.

[3] L. Fermo, D. Occorsio, *Weakly singular linear Volterra integral equations: A Nyström method in weighted spaces of continuous functions*, Journal of Compuational and Applied Mathematics 406 (2022).