# SYMBOL-BASED MULTILEVEL BLOCK $\tau$ PRECONDITIONERS FOR MULTILEVEL BLOCK TOEPLITZ SYSTEMS: GLT-BASED ANALYSIS AND APPLICATIONS

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Preconditioning techniques for Toeplitz linear systems have been a prominent area of research for several decades, particularly in the real symmetric (or, more generally, Hermitian) case, where many effective strategies have been developed. In contrast, the real nonsymmetric case remains less explored, mainly due to the challenges associated with analyzing the eigenvalues and, consequently, the convergence behavior of iterative solvers.

To address this issue, we employ a symmetrization technique that transforms the coefficient matrix into a real symmetric Hankel structure with a known eigenvalue distribution. By leveraging Generalized Locally Toeplitz (GLT) theory, we then develop a novel preconditioning strategy involving centrosymmetric preconditioners, such as those derived from the $\tau$ algebra. This approach constitutes a general framework, as it relies solely on the generating function of the Toeplitz matrix, under the mild assumption that it is well-defined. Moreover, the results can be extended to both the multilevel and the block setting.

Finally, we demonstrate the effectiveness of this approach by applying it to large, dense and ill-conditioned linear systems arising from the discretization of space-fractional diffusion equations. Through a series of numerical experiments, we assess the performance of our proposal against several state-of-the-art preconditioners.