QUADRATURE RULES RELATED TO ORTHOGONALITY ON THE SEMICIRCLE

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Let D_+ be defined as $D_+ = \{z \in \mathbb{C} : |z| < 1, \mathrm{Im} z > 0\}$ and let Γ be a unit semicircle $\Gamma = \{z = \mathrm{E}^{\mathrm{i}\theta} : 0 \leq \theta \leq \pi\} = \partial D_+$. Let $w(z)$ be a weight function which is positive and integrable on the open interval $(-1, 1)$, though possibly singularity at the endpoints, and which can be extended to a function $w(z)$ holomorphic in the half disc D_+. Orthogonal polynomials on the semicircle with respect to the complex-valued inner product

$$(f, g) = \int_\Gamma f(z)g(z)w(z)(\mathrm{i}z)^{-1}\,\mathrm{D}z = \int_0^\pi f(\mathrm{E}^{\mathrm{i}\theta})g(\mathrm{E}^{\mathrm{i}\theta})w(\mathrm{E}^{\mathrm{i}\theta})\,\mathrm{D}\theta$$

was introduced by Gautschi and Milovanović in [2] (for $w(x) = 1$), where the certain basic properties were proved. Such orthogonality as well as the applications involving Gauss-Christoffel quadrature rules were further studied in [1] and [5]. Inspired by Laurie's paper [3], Milosavljević at el. in [4] introduced anti-Gaussian quadrature rules related to the orthogonality on the semicircle, presented some of their properties, and suggested a stable numerical method for their construction. In this lecture we introduce the generalized averaged Gaussian quadrature rules on the semicircle. Two methods for their construction and some properties are included. In addition, the accuracy of such quadrature rules and applications are demonstrated through numerical examples.

References

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