QUADRATURE RULES WITH QUASI-DEGREE OF EXACTNESS

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Many quadrature rules are designed to be exact for easily integrable functions similar to the integrand. The Gauss formula with n nodes is exact on the space of all polynomials of degree $\leq 2n-1$ and it represents a unique optimal interpolatory quadrature rule. It is suitable for application if the integrand is polynomial-like. Note that a function can be similar to certain polynomials, but not similar to some other polynomials. This motivates us to construct a quadrature rule exact only on a subspace of polynomials that share certain properties with the integrand. After choosing m arbitrary points x_k (at which the integrand is defined), we transform the given integral into a sum of an integral that does not cause a quadrature error and an integral with a property that the points x_k are the zeros of its modified integrand. Then, we approximate the integral of the modified integrand by an n-point formula exact on the subspace of polynomials of degree $\leq 2n-1+m$ with m fixed zeros x_k (those fixed zeros are not a disadvantage, since the modified integrand has the same zeros) – such a formula is said to have a quasi-degree of exactness $2n-1+m$.