SIMULTANEOUS GAUSSIAN QUADRATURE

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Suppose you want to integrate one function f with respect to $r > 1$ measures $\mu_1, \ldots, \mu_r$ (or weights $w_1, \ldots, w_r$). The goal is to use N function evaluations and to maximize the degree of exactness. This notion of simultaneous quadrature was introduced by Carlos Borges [1] in 1994. It turns out that the optimal choice is to use the zeros of (type II) multiple orthogonal polynomials as quadrature nodes. These quadrature nodes can be computed as the eigenvalues of a banded Hessenberg matrix and the quadrature weights can be obtained using the left and right eigenvectors of this Hessenberg matrix [2] [3]. The Hessenberg matrix is not symmetric and this causes numerical problems. We show how these can be reduced by transforming the matrix [4]. We will illustrate this by giving some examples.

References

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