ON MATRIX EQUATIONS ASSOCIATED WITH RANDOM WALKS IN THE QUARTER PLANE

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The numerical solution of random walks in the quarter plane leads to solving matrix equations of the kind $X = A_1X^2 + A_0X + A_{-1}$ where A_{-1}, A_0, A_1 are semi-infinite tridiagonal matrices which share the Toeplitz structure everywhere except in the first row. Solving this kind of equations is an important task in the analysis of queuing networks encountered in the applications.

In this talk, we provide an introduction to the problem, present some models from the applications which motivate this analysis, and discuss some algorithmic approaches.

In particular, we provide conditions under which the solution can be written as the sum of a Toeplitz matrix and a compact correction, and present some algorithms which separetly approximate the Toeplitz part and the correction part of the solution. The algorithms which we analyze include fixed point iterations, Newton's method, and the cyclic reduction iteration. Some computational issues are discussed, in particular, solving a Sylvester matrix equation having coefficients with infinite size, representing infinite matrices with a finite number of parameters, and implementing a matrix arithmetic for infinite matrices.