A QUADRATURE METHOD FOR CAUCHY SINGULAR INTEGRAL EQUATIONS WITH ADDITIONAL FIXED SINGULARITIES OF MELLIN TYPE

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This talk deals with a quadrature method for approximating the solutions of Cauchy singular integral equations with additional terms of Mellin convolution type defined as follows

$$a\,u(\tau)+\frac{b}{\pi}\int_{-1}^{1}\frac{u(t)}{t-\tau}dt+\int_{-1}^{1}k(t,\tau)u(t)dt+\int_{-1}^{1}h(t,\tau)u(t)dt=g(\tau),\quad |\tau|<1, \tag{1}$$

where $u(\tau)$ is the unknown, $h(t,\tau)$ and $g(\tau)$ are sufficiently smooth functions, a and b are given real constants such that $a^2+b^2=1$, and $k(t,\tau)$ is a Mellin kernel. The first integral is understood in the Cauchy principal value sense.

Since several mathematical problems in physics and engineering can be reduced to the solution of integral equations of the form (1), the development of numerical methods for approximating their solution has been receiving an increasing interest in recent years. In particular, discretization schemes based on polynomial approximation have been considered in [1, 2, 3], mainly in the case where $k(t,\tau)$ is a special Mellin kernel.

The unknown function u is approximated by a weighted polynomial that is the solution of a finite dimensional equation obtained by discretizing the integral operators by a Gauss-Jacobi quadrature rule. More precisely, in order to achieve stability and convergence results, the Gaussian formula is applied to the Mellin integral operator with a slight modification. The well conditioning of the involved linear systems is proved. The efficiency of the proposed method is shown through some numerical tests.

References

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