GAUSS-KRONROD QUADRATURE FORMULAE BASED ON THE ZEROS OF CHEBYSHEV ORTHOGONAL RATIONAL FUNCTIONS

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Consider the nested sequences of arbitrary complex or infinite poles $\mathcal{A}_k := \{\alpha_j\}_{j=1}^{k}$, $k = 1, 2, \ldots$, outside the interval $I = [-1, 1]$. Let φ_n, $n > 1$ denote the rational function with poles among $\mathcal{A}_n$, and orthogonal to the space of rational functions with poles among $\mathcal{A}_{n-1}$ with respect to the Chebyshev weight function $w(x) = (1-x)^a(1+x)^b$, where $a, b \in \{\pm 1/2\}$. Whenever α_n is real or infinite, the zeros $\{x_{n,k}\}_{k=1}^{n}$ of φ_n are all real, distinct, and inside the interval I; hence, they are the nodes in an n-point rational Gauss-Chebyshev quadrature formula that is exact in the space of rational functions $\tilde{\mathcal{L}}_{2n-1}$ with poles among $\tilde{\mathcal{A}}_{2n-1} := \{\alpha_k, \overline{\alpha}_k\}_{k=1}^{n-1} \cup \{\alpha_n\}$.

In this talk we present $(2n+1)$-point rational Gauss-Kronrod quadrature formulae of the form

$$\int_{-1}^{1} f(x)w(x)dx = \sum_{k=1}^{n} \lambda_{2n+1,k} f(x_{n,k}) + \sum_{j=1}^{n+1} \lambda_{2n+1,n+j} f(y_{n+1,j}) + R_{2n+1}(f)$$

with positive weights $\{\lambda_{2n+1,k}\}_{k=1}^{2n+1}$ and distinct nodes $\{y_{n+1,j}\}_{j=1}^{n+1} \subset I$, interlacing with the nodes $\{x_{n,k}\}_{k=1}^{n}$, that are exact (i.e., $R_{2n+1}(f) = 0$) in a space of rational functions $\hat{\mathcal{L}}_m \supset \tilde{\mathcal{L}}_{2n-1}$ with m as large as possible.