FIRST STEPS TOWARDS THE NUMERICAL QUANTIFICATION OF SOURCE CONDITIONS

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We consider linear ill-posed problems of the form $Ax = y$ with possibly noisy data y and exact solution $x^\dagger$. A classical assumption in the theory of inverse problems are source conditions of the type $x^\dagger \in \mathrm{range}((A^*A)^\mu)$ for some $\mu > 0$. This allows to bound the worst-case error between approximate solutions and $x^\dagger$ as the noise goes to zero, and it yields rules for an appropriate choice of the regularization parameter. In the real-world situation where a fixed operator A and a datum y are given, a good approximation to μ is only available in specific cases, while in general μ is unknown, rendering in particular a-priori parameter choice rules unfeasible. In this talk, we make a first attempt of breaking the disconnection between theory and practice. Based on the Kurdyka-Łojasiewicz inequality and the Landweber method, we develop an algorithm that allows to approximate μ as long as the noise in the data is not too large. We show several numerical examples, including a controlled academical setup where all parameters are available, examples from the RegularizationTools toolbox, and the realistic case where no information about noise or smoothness is available at all.

We also show that there is a simple lower bound for the reconstruction error, which can be computed without any knowledge of a source condition. We again provide numerous numerical examples and explain how the lower bound allows us to better interpret the results of the approximation of μ. It is notable that, if a source condition holds, the lower bound is of the same order as the upper bound.