SCHUR ALGORITHMS FOR MATRIX EQUATIONS

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We consider matrix equations of the type $r(X) = A$, where r is a rational function and A and X are square matrices of the same size. We provide two algorithms for solving the matrix equation, deduced from two different evaluation schemes of the rational function r and more efficient, in term of computational cost, than existing algorithms for the same problem [1].

The algorithms are based on a reduction to (block) triangular matrices using the Schur form, followed by a substitution procedure. For triangular data and unknown, our algorithms for solving the equation have the same asymptotic cost as the evaluation schemes from which they are deduced.

The algorithms are then applied to the computation of primary matrix functions defined by an equation of the type $f(X) = A$, such as the matrix logarithm [2] and the Lambert W function (defined by the equation $X\exp(X) = A$) [3].

References

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[3] M. Fasi, B. Iannazzo, N. J. Higham, *An Algorithm for the Matrix Lambert W Function*, SIAM J. Matrix Anal. Appl., 36(2), 669–685.