ANTI-GAUSSIAN QUADRATURE FORMULAE BASED ON THE ZEROS OF STIELTJES POLYNOMIALS

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It is well known that a practical error estimator for the Gauss quadrature formula is by means of the corresponding Gauss-Kronrod quadrature formula developed by Kronrod in 1964. However, recent advances show that Gauss-Kronrod formulae fail to exist, with real and distinct nodes in the interval of integration and positive weights, for several of the classical measures (cf. [2]). An alternative to the Gauss-Kronrod formula, as error estimator for the Gauss formula, is the anti-Gaussian and the averaged Gaussian quadrature formulae presented by Laurie in 1996 (cf. [1]). These formulae always exist and enjoy the nice properties that, in several cases, Gauss-Kronrod formulae fail to satisfy. Now, it is quite remarkable that for a certain, fairly broad, class of measures, for which the Gauss-Kronrod formulae exist, the anti-Gaussian and averaged Gaussian formulae, based on the zeros of the corresponding Stieltjes polynomials, have elevated degree of exactness, and the estimates provided for the error term of the Gauss formula by either the Gauss-Kronrod or the averaged Gaussian formulae are exactly the same.

References

[1] D. P. Laurie, *Anti-Gaussian quadrature formulas*, Math. Comp., 65 (1996), pp. 739–747.

[2] S. E. Notaris, *Gauss-Kronrod quadrature formulae – A survey of fifty years of research*, Electron. Trans. Numer. Anal. 45 (2016), pp. 371–404.