A GMRES CONVERGENCE ANALYSIS FOR LOCALIZED INVARIANT SUBSPACE ILL-CONDITIONING

G. Sacchi, and **V. Simoncini**
Dipartimento di Matematica,
Università di Bologna,
Piazza di Porta S. Donato, 5,
I-40127 Bologna, Italy
valeria.simoncini@unibo.it

The Generalized Minimal RESidual (GMRES) method is a well established strategy for iteratively solving a large linear system $Ax = b$, where $A \in \mathbb{R}^{n \times n}$ is a nonsymmetric and nonsingular coefficient matrix, and $b \in \mathbb{R}^n$. In the analysis of its convergence for A diagonalizable, a much used upper bound for the relative residual norm involves a min-max polynomial problem over the set of eigenvalues of A, magnified by the condition number of the eigenvector matrix of A. This latter factor may cause a huge overestimation of the residual norm, making the bound non-descriptive in practice. We show that when a large condition number is caused by the almost linear dependence of few of the eigenvectors, a more descriptive analysis of the method's behavior can be performed, irrespective of the location of the corresponding eigenvalues. The new analysis aims at capturing how the GMRES polynomial deals with the ill-conditioning; as a byproduct a new upper bound for the GMRES residual norm is obtained. A variety of numerical experiments illustrates our findings.

References

[1] Giulia Sacchi and Valeria Simoncini. *A new GMRES convergence analysis for localized invariant subspace ill-conditioning*. July 2017. To appear in SIAM J. Matrix Analysis and Appl.