BIORTHOGONAL RATIONAL KRYLOV SUBSPACE METHODS

N. Van Buggenhout, M. Van Barel, and R. Vandebril
Department of Computer Science, KU Leuven
Celestijnenlaan 200A, 3001 Leuven, Belgium
niel.vanbuggenhout@kuleuven.be

Rational Krylov subspace methods use rational functions of a given matrix instead of polynomials to construct a Krylov subspace. The poles of these rational functions can be chosen and allow us to manipulate the convergence of the methods to an area of interest in the spectrum of the matrix.

An overview of known projections of matrices onto various Krylov subspaces is provided. Most notably orthogonal projection onto rational Krylov subspaces, which can be represented by an upper-Hessenberg or inverse upper-Hessenberg matrix pencil. Starting from these results we prove that a tridiagonal matrix pencil suffices to represent the oblique projection of a given matrix onto rational Krylov subspaces. This is the most sparse of several possible representations.

The tridiagonal matrix pencil relates to a six-term recurrence to construct a pair of biorthogonal bases for rational Krylov subspaces. This is a Lanczos-type iteration which elegantly generalizes the polynomial case, in which the recurrence consists of 4 terms. Furthermore this algorithm generalizes, besides Lanczos and biorthogonal Lanczos, several other known algorithms, e.g., AGR/CMV-factorization and more recent results concerning extended Krylov subspaces.

References

[1] N. Van Buggenhout, M. Van Barel, and R. Vandebril, *Biorthogonal Extended Krylov Subspace Methods*, arXiv preprint arXiv:1809.07660, (2018).