QRYLOV

R. Vandebril
Department of Computer Science, KU Leuven
Celestijnenlaan 200A, 3001 Leuven, Belgium.
raf.vandebril@cs.kuleuven.be

In this lecture we will investigate the intimite connection between Krylov subspaces, structured matrices, and the QR algorithm.

We start by revisiting how Krylov subspaces are lurking behind the convergence and the implicit Q theorem in the classical QR algorithm. Both are essential in *understanding Francis's implicitly shifted QR algorithm*. From the QR algorithm, operating on a Hessenberg matrix, it is straightforward to deduce the QZ algorithm, operating on a Hessenberg – upper triangular pair. The implicit QZ algorithm is, like the QR algorithm, a bulge chasing algorithm, with the bulge hopping from one matrix to the other.

Next, we examine extended Krylov subspaces and see that the theory carries over neatly. Instead of a Hessenberg – upper triangular pair we end up with an *extended* Hessenberg – Hessenberg pair. The extended Hessenberg pair is highly structured: the i-th subdiagonal element must be zero in exactly one of the two Hessenbergs. The associated *extended QZ* algorithm is still a bulge chasing/hopping algorithm.

Finally, we discuss rational Krylov subspaces. Now we will have to deal with a Hessenberg – Hessenberg pair, where the poles determining the rational Krylov subspace are encoded in the subdiagonal elements of these Hessenberg matrices. Again we can deduce an implicit Q theorem and develop a *rational QZ* algorithm. We will, however, not be able to chase bulges anymore, instead we will have to manipulate the poles and end up with a pole swapping algorithm. The convergence will be governed by subspace iteration driven by rational functions. Some numerical experiments will reveal the advantages of using this rational QZ algorithm.

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