Strong asymptotics of Nuttall-Stahl polynomials

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Given a germ of an analytic function

$$f(z) = \sum_{k=0}^{\infty} \frac{c_k}{z^{k+1}}$$

which has the analytic continuation along any path in the complex plane which does not go through a finite set of points $f \in \mathcal{A}(\overline{\mathbb{C}} \setminus A), \quad \sharp A < \infty$. Nuttall has put forward the important relation between the *maximal* domain of holomorphicity for the analytic function f and the domain of convergence of the diagonal Padé approximants. The Padé approximants, which are single valued rational functions, approximate a holomorphic branch of the analytic function in the domain of their convergence. At the same time most of the poles of the rational approximants tend to the boundary of the domain of convergence and the support of their limiting distribution models the cuts which make the function f single valued. Nuttall has conjectured (and proved for many important special cases) that these cuts have a minimal logarithmic capacity among all cuts converting the function to a single valued branch. Thus the domain of convergence corresponds to the *maximal* (in the sense of *minimal* boundary) domain of holomorphicity for the analytic function $f \in \mathcal{A}(\overline{\mathbb{C}} \setminus A)$. The complete proof of Nuttall's conjecture (even in a more general setting where the set A has logarithmic capacity 0) was obtained by Stahl. We obtain strong asymptotics for denominators of the diagonal Pade-Approximants for this problem in rather general settings. Our restrictions are that A is a finite set of branch points of f which have the algebro-logarithmic character and which placed at generic positions. The last restriction means that we exclude from our consideration some degenerated "constellations" of the branch points.