Optimal scheduling of the "invigilation" problem

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We define the *invigilation* problem as follows: A university department offers m modules per term, and there are k available invigilators for their exams. Depending on the enrollment, each module j requires w_j invigilators, $w \in \mathbb{N}^m$. Each invigilator i is only available for the exams of certain modules and cannot be assigned more than v_i invigilations per term, $v \in \mathbb{N}^k$. The availability of invigilators to modules can be described as a boolean $k \times m$ matrix E, with $E_{ij} = 1$, if person i can *potentially* be present at the exam of module j, and $E_{ij} = 0$ otherwise.

This model problem represents a number of scheduling applications from diverse fields such as workplace staffing, radio/tv broadcasts, marketing, production processes, etc.

An optimal scheduling solution for the "invigilation" problem is a boolean matrix $X \in \mathbb{N}^{k\times m}$, with $X_{ij} = 1$, if invigilator i is assigned to module j and $X_{ij} = 0$ otherwise. The solution should

$$\text{minimize} \sum_{j=1}^{m} X_{ij}, \quad \text{for } i = 1, 2, \ldots, k,$$

subject to the following constraints:

$$\sum_{j=1}^{m} X_{ij} \le v_i, \text{ for } i = 1, 2, \ldots, k \quad \text{and} \quad \sum_{i=1}^{k} X_{ij} \ge w_j, \quad \text{for } j = 1, 2, \ldots, m.$$

We propose various numerical and heuristic algorithms for the solution of the "invigilation" problem and we investigate their performance on a number of test problems.