Means of structured matrices: properties, applications and algorithms

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Averaging matrices is a problem arising when one has to represent, through a single matrix, the results of several experiments made up by a set of many matrices. Besides the straightforward arithmetic mean, there are other types of means which are suitable for different problems: for positive definite matrices, good averages are obtained by the Karcher mean [1], which verifies all the properties required from a good definition of geometric mean.

In certain applications there is the need to compute means of positive definite matrices which have further structures. A noticeable example arises in radar signal processing, where the matrices to be averaged are correlation matrices, which are Toeplitz and positive definite [2]. Unfortunately, the Karcher mean of Toeplitz matrices is not Toeplitz.

We introduce the new concept of structured geometric mean and prove that it maintains the structure of the input matrices. We restate the properties of geometric mean in terms of the structure and show that most of them are satisfied by our definition. We discuss some applications in which structured means are required. Finally, we provide an iterative algorithm for computing the structured geometric mean and analyze its convergence properties.

References

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[2] F. Barbaresco. *New Foundation of Radar Doppler Signal Processing based on Advanced Differential Geometry of Symmetric Spaces: Doppler Matrix CFAR & Radar Application.* In International Radar Conference 2009, Bordeaux, France, October 2009.