

DISPERSIVE SHOCK WAVES AND UNIVERSALITY

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The short time propagation of long waves as solutions to nonlinear dispersive PDEs is dominated by the nonlinear dynamics versus a small dispersion. Approaching the point of gradient catastrophe (critical point), after which the classical solution becomes multivalued, the dispersion becomes important and the solution starts to develop fast oscillations. This phenomenon is known as dispersive shock wave.

According to Gurevich and Pitaevski, the asymptotic description of the dispersive shock wave within the oscillatory zone is provided by Whitham's analysis. Outside this zone, a long wave is well approximated by the solution to the dispersionless limit of the original equation. However, the general problem of matching the solutions between the two zones is still open. According to Dubrovin's conjecture, the asymptotic behavior around the critical point is "universal" and it is described, at the leading order, by a particular solution to a Painleve' equation. This conjecture has been rigorously proven for the Korteweg-de-Vries equation but should also hold for more general non-integrable dispersive equations.