Soliton reflection in boundary value problems for discrete and continuous nonlinear Schrödinger equations

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Boundary value problems for integrable nonlinear evolution equations cannot be fully linearized in general, except for special kinds of boundary conditions (BCs). On the other hand, linearizable BCs give rise to interesting phenomena. Here I will discuss the nonlinear Schrödinger (NLS) equation on the half line, both in the focusing and the defocusing case (the latter with non-zero BCs at infinity), as well as its integrable discrete counterpart, the Ablowitz-Ladik (AL) lattice. I will show that, in both the continuous and the discrete cases, these problems can be treated by properly extending the potential to the infinite domain and using the inverse scattering transform of the initial value problem. Some BCs can be dealt with simple even and odd extensions. These are the homogeneous Dirichlet and Neumann BCs for NLS as well as their discrete analogues for AL. More general linearizable BCs (such as homogeneous Robin BCs for the NLS and their discrete analogues for AL) can be treated using a special, "mirroring", type of auto-Backlund transformations. In all of these cases, the extension of the problem to the infinite domain is characterized by certain extra symmetries, which impose corresponding symmetries in the scattering coefficients. I will show that, as a result, discrete eigenvalues appear in symmetric pairs (for defocusing NLS), symmetric qua rtets (focusing NLS) or symmetric octets (AL). In turn, these combinations imply that, for every physical soliton, a "mirror" soliton exists, located beyond the boundary, which mediates the reflection experienced by the physical soliton at the boundary. These results provide a nonlinear analogue of the method of images.