Approximate inverse preconditioning for multilevel finite element matrices

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A preconditioner for the conjugate gradient solution of linear systems with block-partitioned coefficient matrices is presented. Such matrices typically arise from multilevel finite element formulations and can be written in a partitioned form

$$\begin{pmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{pmatrix}$$

In multilevel preconditioning, the coupling blocks are usually discarded and *different* preconditioners are chosen for the diagonal blocks. Moreover, it is important to employ a "good" preconditioner for one of these, say K_{aa}, while for K_{bb} a much simpler preconditioner is usually sufficient. The *multilevel* preconditioners have, therefore the following form

$$\begin{pmatrix} M_{aa} & 0 \\ 0 & D_{bb} \end{pmatrix},$$

with D_{bb} being diagonal or block diagonal and we seek to approximate K_{aa} by M_{aa} in an efficient manner. In many situations, $M_{aa} = K_{aa}$ since its size can be remarkably small.

The proposed preconditioner approximates K_{aa}^{-1} and it is derived from a limited memory quasi-Newton procedure with information gathered from the conjugate gradient iterations. Numerical experiments show that it is comparable in terms of convergence and performance with other established multilevel preconditioners. However, since its construction and implementation rely solely on matrix-vector products, the proposed preconditioner is potentially more suitable for parallel processing and/or in situations where the global stiffness matrix is never fully assembled. Results from structural analysis problems are given, tested on quad-core processors and GPUs.