Optimal rational Krylov subspaces for large-scale dynamical systems

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Ruhe's Rational Krylov Subspace is recognized as a powerful tool within Model Order Reduction techniques for linear dynamical systems. However, its success has been hindered by the lack of procedures, which would generate the sequence of shifts used to build the space with good approximation properties.

We begin with the first order passive problems $Au+u_t = 0, \quad u|_{t=0} = b$ for $0 \le t < \infty$, where $u(t), \varphi \in \mathbf{R^N}$ and $A \in \mathbf{R^{N\times N}}$. We will solve this problem by projecting it onto the Rational Krylov Subspace (RKS). We first assume that the numerical range $W(A)$ is known and design a-priori algorithms of optimal shift generation. We consider this problem in the frequency domain and reduce it to the third Zolotaryov problem in complex plane.

Then we propose a recursive greedy algorithm for adaptive choice of shifts taking into account non-uniformity of the spectrum. The algorithm is based on an explicit formula for the residual in the frequency domain allowing adaptive shift optimization at negligible cost.

Finally, we extend the our approach from the first order problem to the solution of passive high and infinite order dynamical systems.

We illustrate obtained results with application to first order and fractional diffusion Maxwell's system.