Computation of Jacobi matrices of uncountable systems of iterated functions and the solution of an inverse problem

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Consider the random process in $\mathbf{R}$ given by $x_{n+1} = \delta(x_n - \beta) + \beta$, where $0 < \delta < 1$ is a real parameter and β is drawn randomly from a probability distribution $\sigma(\beta)$. Clearly, the points x are *attracted* towards the points β and, in the limit, their distribution converges to a measure μ. This latter is the invariant measure of a system of Iterated Functions (I.F.S.).

Usually in I.F.S. theory, the distribution $\sigma(\beta)$ is discrete and composed of a finite number of atoms. In this talk, we allow σ to be any compactly supported distribution. We describe a new technique for computing the Jacobi matrix of the measure μ, that is numerically stable for matrix orders as large as hundreds of thousands. The same theory can be reversed into an efficient technique for solving an inverse reconstruction problem, that requires to find σ from the knowledge of μ. This problem can be cast in the form of a generalized Gaussian integration problem. Introductory results to the new material presented here can be found in [1, 2].

References

[1] G. Mantica, *A Stieltjes Technique for Computing Jacobi Matrices Associated With Singular Measures*, Constr. Appr., 12 (1996), pp. 509–530.

[2] G. Mantica, *Polynomial Sampling and Fractal Measures: I.F.S.–Gaussian Integration*, Num. Alg. 45 (2007), pp. 269–281; *Dynamical Systems and Numerical Analysis: the Study of Measures generated by Uncountable I.F.S*, Num. Alg. 55 (2010), pp. 321–335.