Adaptive Filon methods for the computation of highly oscillatory integrals

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First we revisit some important families of quadrature methods (Filon type methods and exponentially fitted methods) for highly oscillatory integrals, all sharing the remarkable property that their accuracy drastically improves as the frequency grows and we apply these methods to integrals of the form $\int_0^h f(x)e^{\mathrm{i}\omega x}dx, \omega > 0$. We also show that, for this type of problem, the methods are strongly related.

The EF rules depend upon frequency dependent nodes that start off at the Gauss nodes when the frequency is zero and end up at the Lobatto nodes when the frequency tends to infinity. This makes the rules well suited for small and very large frequencies. However, for a particular frequency of moderate size, the computation of the nodes is expensive (due to ill-conditioning and iteration).

On the other side, the Filon-type rules with (fixed, i.e. frequency independent) Lobatto nodes behave very well for large frequencies, but not so good for smaller frequencies, because Lobatto-type methods are of lower classical order than Gauss-type methods.

What we propose in this talk is a new type of quadrature rules with frequency dependent nodes, for which the evaluation for a particular value of the frequency is cheap, and which is suited for small as well as large frequencies. The ill-conditioning and the need for iteration is removed by the introduction of some S-shaped functions.

Finally a technique is proposed that can be used to produce accurate error estimates, allowing a successful practical implementation of the quadrature rules discussed.