Numerical solution of an integro-differential equation modelling the neuronal activity

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Abstract

Modelling the neuronal activity in the cerebral cortex is a very challenging task, which has nowadays multiple applications not only in Medicine (interpretation of data, such as EEG, fMRI and optical imaging) but also in Robotics. One of the most promising approaches in this domain are the so called Neural Field Equations (NFE), where a certain region of the cortex is considered as a *continuous field of neurons* with certain electrical properties. In this work we consider a two-dimensional NFE in the form

$$c\frac{\partial}{\partial t}V(\bar{x},t) = I(\bar{x},t) - V(\bar{x},t) + \int_{\Omega} K(|\bar{x}-\bar{y}|)S(V(y,t-\tau(\bar{x},\bar{y}))d\bar{y}, \qquad (1)$$

$\bar{x} \in \Omega \subset R^2, t \in [0,T]$, where the unknown $V(\bar{x},t)$ is a continuous function $V : \Omega \times [0,T] \to R$, I, K and S are given functions; c is a constant. We search for a solution V of this equation which satisfies the initial condition $V(\bar{x},t) = V_0(\bar{x},t), \quad \bar{x} \in \Omega, \quad t \in [-\tau_{max}, 0]$, where $\tau_{max} = \max_{\bar{x},\bar{y}\in\Omega} \tau(\bar{x},\bar{y})$. Here τ is a delay depending on $\bar{x}$ and $\bar{y}$ (as a particular case, we also consider the case $\tau \equiv 0$).

Equation (1) without delay was introduced first by Wilson and Cowan [3], and then by Amari [1], to describe excitatory and inhibitory interactions in populations of neurons.

We describe a numerical method recently introduced [2] to approximate the solution of equation (1). The accuracy and efficiency of the method are discussed and some numerical examples are presented which illustrate its performance.

This talk is based in a joint work with E. Buckwar, from the University of Linz.

References

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