Fractional diffusion problems

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Abstract

Fractional derivatives have been used to model anomalous diffusion and therefore the fractional diffusion equation has been presented as a suitable model for many problems that appear in different fields, such as biology, finance and hydrology. This equation can be obtained from the standard diffusion equation by replacing the second order spatial derivative by a fractional operator of an order between one and two. This operator is defined by a combination of the left and right Riemann-Liouville derivatives. The most simple problems involving Riemann-Liouville derivatives lead to challenging questions, from different point of views, such as, mathematical or physical and in this talk we will discuss some of them. We start with the development of a numerical method for a diffusion equation defined in an open domain and show some numerical solutions to observe the anomalous diffusive process. Then, we present some interesting properties of the Riemann-Liouville derivative to discuss a few of the challenges we are faced with when we want to define a well-posed problem in a bounded domain, subject to Dirichlet boundary conditions. We also analyse the additional difficulties encountered related to the numerical discretisation of this operator.