



## VDM60

### Nonlinear Evolution Equations and Linear Algebra

A conference to celebrate the 60th birthday of  
*Cornelis van der Mee*

University of Cagliari, Italy  
September 2–5, 2013





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## Scientific Program

- **Plenary Speakers**

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- **Tommaso Ruggeri**, University of Bologna, Italy

- **Sessions**

1. **Nonlinear Evolution Equations**
2. **Kinetic Theory, Transport Theory, and Applications**
3. **Numerical Linear Algebra and Applications**
4. **Poster Session**





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## **Keynote Speakers**

## GEOMETRIC MEANS OF MATRICES: ANALYSIS AND ALGORITHMS

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Matrix geometric means provide a tool to average a set of positive definite matrices in such a way that the inverse of the matrix mean coincides with the mean of the inverse matrices. Different definitions of matrix mean have been given in the literature together with algorithms for their computations.

In some applications one needs to compute the geometric mean of a set of structured matrices. This happens, for instance, in radar detection problems where the matrices to average are Toeplitz. For physical reasons, one requires that the mean of structured matrices maintains the same structure of the input matrices. Unfortunately this requirement is not satisfied by the available definitions.

In this talk we give an overview on matrix geometric means, and recall their relationships with the Riemannian geometry of the cone of positive definite matrices. Then we treat with more attention the Karcher mean, relate it to a matrix equation, and provide numerical algorithms for its solution. Finally we present a modified version of the Karcher mean which preserves the structure of the input matrices and satisfies almost all the nice properties of the scalar geometric mean. An effective numerical algorithm for its computation is given in terms of solution of a vector equation.

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## RATIONAL SOLITONS OF RESONANT WAVE INTERACTION MODELS

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Integrable models of resonant interaction of two or more waves in 1+1 dimensions are known to be of applicative interest in several areas. Here we consider a system of three coupled wave equations which includes as special cases the vector nonlinear Schrödinger equations and the equations describing the resonant interaction of three waves. The Darboux construction of soliton solutions is applied with the condition that the solutions have rational, or mixed rational-exponential, dependence on coordinates. Our algebraic construction relies on the use of nilpotent matrices and their Jordan form.

## SPECTRAL PROBLEMS ASSOCIATED WITH THE MATRIX-VALUED AKNS EQUATION

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We will discuss spectral problems arising in the study of the matrix-valued AKNS system. The main focus will be on results that can be obtained under minimal local assumptions and for matrix coefficients that have nonvanishing and different asymptotics as  $x \rightarrow \pm\infty$ . In addition to giving a partial review of prior work on this subject we will describe some recent results and problems that might warrant further study.

## RECENT RESULTS IN RATIONAL EXTENDED THERMODYNAMICS: MACROSCOPIC APPROACH AND MAXIMUM ENTROPY PRINCIPLE FOR DENSE AND RAREFIED POLYATOMIC GASES

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After a brief survey on the principles of Rational Extended Thermodynamics of monatomic gas (entropy principle, constitutive equations of local type, symmetric hyperbolic systems, main field, principal sub-system) we present in this talk a recent new approach to deduce hyperbolic system for dense gases not necessarily monatomic.

In the first part of the talk we study extended thermodynamics of dense gases by adopting the system of field equations with a different hierarchy structure to that adopted in the previous works. It is the theory of 14 fields of mass density, velocity, temperature, viscous stress, dynamic pressure and heat flux. As a result, all the constitutive equations can be determined explicitly by the caloric and thermal equations of state as in the case of monatomic gases. It is shown that the rarefied-gas limit of the theory is consistent with the kinetic theory of gases.

In the second part, we limit the result to the physically interesting case of rarefied polyatomic gases and we show a perfect coincidence between ET and the procedure of Maximum Entropy Principle. The main difference with respect to usual procedure is the existence of two hierarchies of macroscopic equations for moments of suitable distribution function, in which the internal energy of a molecule is taken into account.

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# **1. Session on Nonlinear Evolution Equations**

SCALAR AND VECTOR NONLINEAR SCHRÖDINGER SYSTEMS WITH NON-ZERO BOUNDARY  
CONDITIONS

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Despite having been intensely investigated over the last forty years, nonlinear Schrödinger (NLS) systems still offer many surprises. In this talk we discuss recent results on both focusing and defocusing, both scalar and vector, NLS equations with non-zero boundary conditions at infinity. A number of explicit soliton solutions will be discussed, as well as spectral problems for special classes of initial conditions.

## INTEGRABLE FLOWS FOR STARLIKE CURVES IN CENTROAFFINE SPACES

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We construct integrable hierarchies of flows for curves in centroaffine  $\mathbb{R}^3$  through a natural pre-symplectic structure on the space of closed unparametrized starlike curves. We show that the induced evolution equations for the differential invariants are closely connected with the Boussinesq hierarchy, and prove that the restricted hierarchy of flows on curves that project to conics in  $\mathbb{R}P^2$  induces the Kaup-Kuperschmidt hierarchy at the curvature level.

This is joint work with Tom Ivey (College of Charleston), and Gloria Marí Beffa (University of Wisconsin-Madison).

### References

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## COMPUTATION OF RELEVANT SCATTERING DATA IN THE ZAKHAROV-SHABAT SYSTEM

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In the numerical solution of non-linear PDEs of integrable type by means of the Inverse Scattering Transform technique, the first step consists of identifying the relevant scattering parameters of the associated Zakharov-Shabat system. In particular, it is important to identify both the bound state parameters with their multiplicities, and the so-called norming constants.

To this end, it is necessary to compute the coefficients  $\{c_{js}\}_{j=1, s=0}^{M, n_j-1}$  and the parameters  $\{f_j\}_{j=1}^M$  of a monomial-exponential sum of the type

$$h(x) = \sum_{j=1}^M \sum_{s=0}^{n_j-1} c_{js} x^s e^{f_j x},$$

where  $M$  and  $\{n_j\}_{j=1}^M$  are positive integers and  $\{c_{js}\}_{j=1, s=0}^{M, n_j-1}$  and  $\{f_j\}_{j=1}^M$  are complex or real parameters with  $c_{j, n_j-1} \neq 0$ , given  $2N$  sampled data  $h(k)$  for  $k = k_0, k_0 + 1, \dots, k_0 + 2N$  with  $k_0 \in \mathbb{N}^+ = \{0, 1, 2, \dots\}$  and  $N \geq L = n_1 + \dots + n_M$ .

In this talk we illustrate a linearization technique to numerically solve this non-linear approximation problem. It is based on the following steps:

1. Identification of the common rank of two square Hankel matrices  $H_0$  and  $H_1$  of order  $N$  generated by the  $2N$  given data;
2. Computation of the parameters  $M$ ,  $\{n_j\}_{j=1}^M$  and  $\{f_j\}_{j=1}^M$  by solving a generalized eigenvalue problem;
3. Computation of the coefficients  $\{c_{js}\}_{j=1, s=0}^{M, n_j-1}$  by solving an overdetermined linear system.



GLOBAL NORMAL FORMS AND SPECTRAL PROPERTIES FOR PERTURBATIONS OF HARMONIC  
OSCILLATORS

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We outline some recent investigations on global normal forms and spectral properties problems second order linear differential operators which might be viewed as perturbations (not necessarily self-adjoint) of multidimensional anisotropic harmonic oscillators  $H = -\Delta + \sum_{j=1}^n \omega_j x_j^{2k_j}$ ,  $\omega_j \in \mathbb{C}$ ,  $\operatorname{Re} \omega_j > 0$ ,  $k_j \in \mathbb{N}$ ,  $j = 1, \dots, n$ .

The results are obtained in collaboration with G. Trnquilli (Università di Cagliari).

## AUTOMORPHIC LIE ALGEBRAS

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Automorphic Lie Algebras are Lie algebras tensored with homogeneous functions of the spectral parameter, such that the elements are invariant under the combined action of a Platonic group acting via two irreducible representations on both the original simple Lie algebra and the spectral parameter (e.g. [1], [2]). They have been introduced in the context of algebraic reduction of integrable systems (Lax pairs), but they turn out to be very interesting in their own right, they show much more structure than originally anticipated and they can be described almost completely independent of the chosen group. The presence of a modular invariant complicates the analysis of the structure theory along the lines of the classical classification theory of complex Lie algebras, but final results are now becoming visible and this talk will report on the latest developments [3].

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# BREAKING MECHANISM FROM A VACUUM POINT IN THE DEFOCUSING NONLINEAR SCHROEDINGER EQUATION

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We study the wave breaking mechanism for the weakly dispersive defocusing nonlinear Schroedinger (NLS) equation with a constant phase dark initial datum that contains a vacuum point at the origin. We prove by means of the exact solution of the initial value problem that, in the dispersionless limit, the vacuum point is preserved by the dynamics until breaking occurs at a finite critical time. In particular, both Riemann invariants experience a simultaneous breaking at the origin. Although the initial vacuum point is no longer preserved in the presence of a finite dispersion, the critical behavior manifests itself through an abrupt transition occurring around the breaking time.

SYLVESTER EQUATIONS AND INTEGRABLE SYSTEMS: A BIDIFFERENTIAL CALCULUS  
PERSPECTIVE

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Matrix Sylvester equations frequently show up in connection with soliton solutions of integrable partial differential and difference equations. Special solutions are Cauchy-like matrices. In the bidifferential calculus approach, matrix Sylvester equations emerge from a quite general result about binary Darboux transformations (A. Dimakis and F. Mueller-Hoissen, SIGMA 9 (2013) 009). We recall this result and present several examples.

## EFFECTS OF INERTIA AND STRATIFICATION IN INCOMPRESSIBLE IDEAL FLUIDS: PRESSURE IMBALANCES BY RIGID CONFINEMENT

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This talk will be principally addressed on the inertial properties of an incompressible Euler two dimensional fluid filling a horizontal channel and in hydrostatic equilibrium at infinity. The interplay between action-at-a-distance, incompressibility and constraints can lead to non-conservation of horizontal momentum even if there are no external horizontal forces acting on the system. The variation of density along the boundaries affects the evolution of the total vorticity of the fluid. The results of Euler equations obtained for small density variations will be compared with long-wave asymptotic models which provide closed-form mathematical expressions for more general results.

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# INTEGRABLE MULTIDIMENSIONAL PDES OF HYDRODYNAMIC TYPE: METHOD OF SOLUTION AND MULTIDIMENSIONAL WAVE BREAKING

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Integrable PDEs of hydrodynamic type, including physically relevant examples like the dispersionless Kadomtsev - Petviashvili, the Boyer - Finley and the heavenly equations, arise from the commutation of vector fields and can be studied using a novel Inverse Spectral Transform [1, 2]. In particular, the nonlinear Riemann - Hilbert inverse problem is a powerful tool i) to study the longtime behavior of localized solutions, ii) to establish if such solutions break, due to the lack of dispersion and dissipation, and, if they do, to extract the analytic features of such a breaking in a surprisingly explicit way; iii) to construct distinguished examples of exact implicit solutions [3]. A summary of the above theory is presented, together with some recent results on the rigorous aspects of such a theory, obtained in collaboration with P. G. Grinevich and D. Wu. This presentation is dedicated to Manakov's memory.

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## MULTIPLE-POLE SOLUTIONS OF THE NONLINEAR SCHRÖDINGER EQUATION

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We will start by a short resume on an operator theoretic approach to the Nonlinear Schrödinger equation, with the aim to motivate a solution formula which gives a unified access to the multiple-pole solutions. The main result is a complete asymptotic description of these solutions, which was so far only achieved for cases of low complexity by Olmedilla. After an overview on the geometric and algebraic ingredients of the proof, we will conclude by a discussion of cases of higher degeneracy and a comparison with the situation for the KdV equation.

# ANALYTICAL APPROXIMATIONS OF THE NONLINEAR SCHRÖDINGER EQUATION: APPLICATIONS TO OPTICAL COMMUNICATIONS AND INFORMATION THEORY

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The propagation of light in fiber-optic links is governed by the nonlinear Schrödinger equation with variable coefficients. Efficient numerical integration algorithms and analytical models for the evolution of the statistical properties of a stochastic signal are the main ingredients to solve several fundamental problems in the field of optical communication and information theory [1, 2].

Here, we introduce some approximated solutions of the equation and discuss their accuracy, complexity, and possible applications. In particular, as a working example, we consider the evaluation of the maximum information rate that can be reliably transmitted through a nonlinear fiber-optic channel [1, 3].

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## PROPAGATION AND CONTROL OF NANOSCALE MAGNETIC-DROPLET SOLITONS

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Recent results on propagating, solitary magnetic wave solutions of the Landau-Lifshitz equation with uniaxial, easy-axis anisotropy in thin (two-dimensional) magnetic films will be illustrated. These localized, nontopological wave structures, parametrized by their precessional frequency and propagation speed, extend the stationary, coherently precessing “magnon droplet” to the moving frame, a non-trivial generalization due to the lack of Galilean invariance. Propagating droplets move on a spin wave background with a nonlinear droplet dispersion relation that yields a limited range of allowable droplet speeds and frequencies. The droplet is found to propagate as a Nonlinear Schroedinger bright soliton in the weakly nonlinear regime [1]. Using spin transfer torque underneath a nanocontact on a magnetic thin film with perpendicular magnetic anisotropy (PMA), the generation of dissipative magnetic droplet solitons was announced this year for the first time, following its theoretical prediction [2]. Rich dynamical properties (including droplet oscillatory motion, droplet spinning, and droplet breather states) have been experimentally observed and reported. After reviewing the conservative magnetic droplet, some properties of the soliton in a lossy medium will be discussed [3]. In particular, it will be shown that the propagation of the dissipative droplet can be accelerated and controlled by means of an external magnetic field. Soliton perturbation theory corroborated by two-dimensional micromagnetic simulations predicts several intriguing physical effects, including the acceleration of a stationary soliton by a magnetic field gradient, the stabilization of a stationary droplet by a uniform control field in the absence of spin torque, and the ability to control the solitons speed by use of a time-varying, spatially uniform external field. Soliton propagation distances approach 10  $\mu\text{m}$  in low-loss media, suggesting that droplet solitons could be viable information carriers in future spintronic applications, analogous to optical solitons in fiber optic communications.

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## A KELLER-SEGEL MODEL IN CHEMOTAXIS WITH BLOW-UP SOLUTIONS

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We study the Neumann initial-boundary value problem for the fully parabolic Keller-Segel type system [1] with time dependent coefficients

$$\begin{aligned}u_t &= \Delta u + k_1(t) \operatorname{div}(u \nabla v), \quad x \in \Omega, t \in (0, t^*), \\v_t &= k_2(t) \Delta v - k_3(t) v + k_4(t) u, \quad x \in \Omega, t \in (0, t^*), \\ \frac{\partial u}{\partial n} &= \frac{\partial v}{\partial n} = 0, \quad x \in \partial \Omega, t \in (0, t^*), \\u(x, t) &= u_0(x), \quad v(x, t) = v_0(x), \quad x \in \Omega,\end{aligned}$$

where  $\Omega$  is a bounded domain in  $R^N$  with smooth boundary,  $\frac{\partial}{\partial n}$  is the normal derivative on the boundary and  $t^*$  is the blow up time. This system forms the core of numerous models used in mathematical biology to describe the spatio-temporal evolution of cell populations governed by both diffusive migration and chemotactic movement towards increasing gradients of a chemical that they produce themselves (chemotaxis). We derive conditions on the data and geometry of  $\Omega$ , sufficient to obtain an explicit lower bound for the blow-up time.

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# THE INVERSE SCATTERING TRANSFORM FOR THE DEFOCUSING NONLINEAR SCHRÖDINGER EQUATION WITH NONZERO BOUNDARY CONDITIONS

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We have developed a rigorous theory of the inverse scattering transform for the defocusing nonlinear Schrödinger equation with nonvanishing boundary values  $q_{\pm} \equiv q_0 e^{i\theta_{\pm}}$  as  $x \rightarrow \pm\infty$ . The direct problem is shown to be well-posed for potentials  $q$  such that  $q - q_{\pm} \in L^{1,2}(\mathbb{R}^{\pm})$ , for which analyticity properties of eigenfunctions and scattering data are established. The inverse scattering problem is formulated and solved both via Marchenko integral equations, and as a Riemann-Hilbert problem in terms of a suitable uniform variable. The asymptotic behavior of the scattering data is determined and shown to ensure the linear system solving the inverse problem is well-defined. Finally, the triplet method is developed as a tool to obtain explicit multisoliton solutions by solving the Marchenko integral equation via separation of variables.

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**2. Session on  
Kinetic Theory, Transport Theory,  
and Applications**

## SIGNAL-NOISE INTERACTION IN NONLINEAR OPTICAL FIBERS: A FLUID-DYNAMIC APPROACH

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We consider the one-dimensional NLSE for optical fibers under noisy input conditions. Assuming small dispersion, we (formally) approximate the NLSE with a “semiclassical” fluid-dynamic system of Madelung type. Then, assuming high signal-to-noise ratio, a perturbative procedure is applied to the Madelung system in order to study the propagation of a deterministic signal affected by a band-limited white noise.

## SOME NEW RESULTS IN GEOMETRICAL OPTICS

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In this talk I want to present some recent results obtained in the framework of Geometrical Optics from the inverse point of view. We shall be concerned with the propagation of light in a continuous transparent inhomogeneous and isotropic medium, dispersive or not. We put and solve the two following inverse problems of geometrical optics:

1) **3-dimensional inverse problem:** Given a two-parametric family of curves  $\mathcal{F}_2: f(x, y, z) = c_1, g(x, y, z) = c_2$ , inside a 3-dimensional medium  $\mathcal{M}_3$ , we want to find the refractive-index distributions  $n(x, y, z)$  allowing for the creation of the given family of curves as a family of monochromatic light rays.

2) **2-dimensional inverse problem:** Given a monoparametric family of curves  $\mathcal{F}_1$ : inside a 2-dimensional medium  $\mathcal{M}_2$ , lying on a regular surface  $S$ , we want to find the refractive-index distributions  $n = n(u, v)$  allowing for the creation of the given family of curves as a family of monochromatic light rays. Our main results are:

**Proposition 1:** Given a family  $\mathcal{F}_2$  lying on a medium  $\mathcal{M}_3$ , all the refractive-index distributions  $n(x, y, z)$  allowing for the creation of the given family of curves as a family of monochromatic light rays, are solutions of the system of two first order linear PDE:  $\alpha n_x - n_y + \Omega_1 n = 0, \beta n_x - n_z + \Omega_2 n = 0$ , in the unique unknown function  $n(x, y, z)$  where  $\alpha(x, y, z), \beta(x, y, z), \Omega_1(x, y, z), \Omega_2(x, y, z)$  are functions depending only on the given family of light rays.

**Proposition 2:** Given a family  $\mathcal{F}_1$ , inside a medium  $\mathcal{M}_2$  lying on a regular surface  $S$ , with a line element given by  $ds^2 = Edu^2 + 2Fdudv + Gdv^2$ , all the refractive-index distributions  $n(u, v)$  allowing for the creation of the given family of curves as a family of monochromatic light rays, are solutions of the linear first order PDE:  $(G - \gamma F)n_u - (F - \gamma E)n_v + \Omega n = 0$ , in the unknown function  $n(u, v)$ , where  $\gamma = \frac{f_v}{f_u}$  is a function of  $u, v$  depending only on the given family;  $E, F, G$  are the coefficients of the assigned metric on  $S$ , and  $\Omega$  is a functions of  $u, v$  depending both of the family and on the metric.

## NONLINEAR ANALYSIS OF THE TWO-MASS-SKATE BICYCLE MODEL

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A simplified model of bicycle, called two-mass-skate (TMS), was recently developed by Kooijmann et al. [3] to show that the self-stability of a bicycle does not depend on neither gyroscopic nor caster effects.

In this paper, we improve this model by revising its kinematics and by imposing no restrictions on the geometry of the rear and front frames, that is, we consider their distributed masses. Furthermore, we assume that the two point wheels have masses without moments of inertia, thus, the trail is always zero.

Taking the nonholonomic constraints on the velocities into account, we then derive the nonlinear equations of motion for the system from a geometric point of view [1], [2]. Further, studying the behaviour of this system, we analyze its stability, which exhibits some peculiar aspects due to the non-holonomy of the problem.

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## ANALYTICAL MECHANICS OF A RELATIVISTIC PARTICLE IN A POSITIONAL POTENTIAL

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We discuss the Lagrangian and Hamiltonian formulation of the dynamics of a classical particle subject to a potential that depends only on its position. In particular, consistency requires that time reparametrization invariance be preserved. First quantization of the model leads to a Klein-Gordon equation coupled to a scalar potential. The approach proposed in this letter can be useful in the study of phenomenological models where the potential is not derived from first principles.

## EXTENDED THERMODYNAMICS FROM THE LAGRANGIAN VIEW-POINT

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Extended Thermodynamics (E.T.) is a powerful and well established theory (see [1] as example). It leads to first order quasi-linear symmetric hyperbolic systems of field equations, guarantees the well-posedness of initial value problem and finite speeds of propagation. Usually it is formulated from the Eulerian view-point: For every point  $\underline{x}$  and time  $t$ , attention is focused to the physical properties of the material particle transiting through that position at the time  $t$ . In this talk it will be shown how the same considerations may be followed from the Lagrangian view-point: Attention is focused to each material particle and to its physical properties, during all the motion of the same particle.

The conservation laws of mass, momentum and energy, from the Lagrangian view-point, have already been treated in literature (see, for example, the textbook [2] from page 64). Here a similar procedure is followed for all the balance laws of E.T. with an arbitrary number of moments.

It is also shown how the Galilean Relativity Principle and some symmetry condition, which are present in the Eulerian view-point, can be "translated" in the Lagrangian view-point, where they are no more so self-evident.

This treatment may be applied to many possible physical situations, for example to semi-conductors, to E.T. of a moving surface and of an extensible wire.

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**3. Session on  
Numerical Linear Algebra  
and Applications**

## DECAY PROPERTIES FOR FUNCTIONS OF MATRICES OVER $C^*$ -ALGEBRAS

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We extend existing results on the off-diagonal decay of the entries of analytic functions of banded and sparse matrices to the case where the matrix entries are elements of a  $C^*$ -algebra. For instance, the matrix entries could be bounded linear operators on a Hilbert space or continuous complex-valued functions on a compact Hausdorff space. The main ingredients are classical approximation theory and the holomorphic functional calculus.

The case of quaternionic matrices will also be discussed, together with possible applications.

## CHARACTERISTIC CURVES IN MODELING OF THE EARTH CRUST AND UPPER MANTLE

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Nearly all applied in geophysics and seismology mathematical models of the Earth crust and upper mantle are based on representation of our planet as non-elastic solid body. The wave propagation in such body is described by a system  $S$  of three strongly coupled linear hyperbolic equations. Following the standard geophysical approach, Earth structure is locally modeled as a half space with jump discontinuities, and therefore the coefficients in  $S$  are piece-wise constant functions. In this talk is considered one geometrical approach to the solutions of system  $S$ . It is based on the geometric properties of characteristic curves of  $S$  and Geometrical optics is used to compute reflection and refraction of the characteristics at the discontinuities. This approach allows reasonable 3-D modeling of the Earth structure and implementing of algorithms for 3-D inverse problem in geophysics

## REGULARIZED NONCONVEX MINIMIZATION FOR IMAGE RESTORATION

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Tikhonov regularization for the linear equation  $Ax = y$  involves the minimization of a convex functional of type  $\Phi(x) = Q(Ax - y) + \alpha G(x)$ , where  $Q$  measures the residual term  $Ax - y$ ,  $G$  is a penalty function incorporating the solution  $x$  or its derivatives, and  $\alpha$  is the regularization parameter. In particular, both  $Q(\cdot)$  and  $G(\cdot)$  are the square of the L2-norm,  $\|\cdot\|_2^2$ , in the simplest case of classical Tikhonov regularization in Hilbert spaces.

In this talk, we discuss, in Banach spaces setting, a regularization functional  $\Phi$  whose penalty term  $G$  depends on the model operator  $A$ , as introduced in [2] for Hilbert spaces. Furthermore, we solve the associated minimization problem by an iterative approach based on a variant of the Landweber method [1]. To speed up the iterations, otherwise too slow, a modification of the penalty term is used, leading to a nonconvex functional.

The obtained nonlinear algorithm has been applied to the linear problem of image deblurring, the removal of blur and noise from a digital image.

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## A MODULARITY-BASED SPECTRAL GRAPH ANALYSIS

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The modularity matrix of a graph is a special rank-one modification of the adjacency matrix, introduced by Newman and Grivan in the framework of graph clustering and community detection problems, see e.g., [1]. In fact, eigenvectors of modularity matrices can be exploited in community detection algorithms in the same way as eigenvectors of Laplacian matrices are currently utilized for solving graph partitioning or bandwidth reduction problems.

We perform an in-depth spectral analysis of modularity matrices. In particular, we prove certain properties of nodal domains induced by eigenvectors of modularity matrices, analogous to those known for graph Laplacian matrices, see e.g., [2]; and we outline the relationship between eigenvalues of modularity matrices and certain combinatorial descriptions of tightly connected subgraphs.

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## FAST RANKING OF NODES ON DIGRAPHS

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One of the main issues in complex networks theory is to find the “most important” nodes within a graph  $G$ . To this aim, one can use matrix functions applied to its adjacency matrix. We will introduce a new computational method to rank the nodes of a directed unweighted network according to the values of these functions. The algorithm uses a partial singular value decomposition, in order to obtain a low-rank approximation of the adjacency matrix, and then Gauss quadrature is used to refine the computation. The method is compared to other approaches on networks coming from real applications, e.g. in software engineering, bibliometry and social networks.



## FREDHOLM INTEGRAL EQUATIONS ON THE REAL SEMIAXIS: A NUMERICAL METHOD

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In this talk we consider Fredholm integral equations of the form

$$f(x) + \mu \int_0^{\infty} k(x, y) f(y) w(y) dy = g(x), \quad x \in (0, \infty), \quad (1)$$

where  $w(y) = e^{-\frac{1}{y^\alpha} - y^\beta}$ ,  $\alpha > 0$ ,  $\beta > 1$ ,  $\mu \in \mathbb{R}$ , and the given functions  $k$  and  $g$  are continuous and exponentially monotonic at the endpoints of the interval  $(0, \infty)$ .

We approximate the solution of (1) by using a Nyström method, which we prove to be stable and convergent. The theoretical background of the method (i.e. the main difficulty) is the construction of new function spaces connected to the weight  $w$  and the related estimates of the error of best polynomial approximation (see [1]).

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## ON THE SOLUTION OF CERTAIN ALGEBRAIC RICCATI EQUATIONS ARISING FLUID QUEUES

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Consider a nonsymmetric algebraic Riccati equation (NARE)  $C + XA + DX - XBX = 0$ , where the unknown  $X$  has size  $m \times n$ , and where the coefficients  $A$ ,  $B$ ,  $C$  and  $D$  are real  $n \times n$ ,  $n \times m$ ,  $m \times n$  and  $m \times m$  matrices, respectively. Assume that the block coefficients are such that  $M = \begin{bmatrix} A & -B \\ C & D \end{bmatrix}$  is a nonsingular M-matrix or a singular irreducible M-matrix. The solution  $X$  of interest is the minimal nonnegative.

In the modeling of an adaptive  $MMA P[K]/PH[K]/1$  queue, the matrix  $D$  is a  $K \times K$  block diagonal matrix with  $h \times h$  blocks. We present a new algorithm for computing the minimal nonnegative solution of the NARE where we exploit the structure of the matrix  $D$ . The solution of the original NARE is computed by solving a set of correlated NAREs, with coefficients of small size, obtained by a suitable block partitioning of the coefficients  $A$ ,  $B$ ,  $C$  and  $D$ . More specifically, the sought solution  $X$  is partitioned in blocks  $X_i$ ,  $i = 1, \dots, K$ , and  $X_i$  is the minimal nonnegative solution of the  $i$ -th NARE

$$C_i + X\tilde{A}_i + D_iX - XB_iX = 0, \quad \tilde{A}_i = A - \sum_{j=1, j \neq i}^K B_jX_j, \quad (2)$$

for  $i = 1, \dots, K$ , where the coefficient  $\tilde{A}_i$  of the above equation depends on the solutions  $X_j$ , with  $j \neq i$ , of the remaining  $K - 1$  NAREs. To solve the above equations we propose an iterative scheme, where we replace the unknown coefficient  $\tilde{A}_i$  with an approximation. Convergence results are proved by using properties of nonnegative matrices and M-matrices. From the analysis of the computational cost and from the numerical experiments, the algorithm is more effective than the standard algorithms when the block size  $K$  is larger than  $h$ .

## A MODIFIED TSVD METHOD FOR DISCRETE ILL-POSED PROBLEMS

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Truncated singular value decomposition (TSVD) is a popular method for solving linear discrete ill-posed problems with a small to moderately sized matrix  $A$ . Regularization is achieved by replacing the matrix  $A$  by its best rank- $k$  approximant, which we denote by  $A_k$ . The rank may be determined in a variety of ways, e.g., by the discrepancy principle or the L-curve criterion. In this talk, we present a novel regularization approach, in which  $A$  is replaced by the closest matrix in a unitarily invariant matrix norm with the same condition number as  $A_k$ . Numerical examples illustrate that this regularization approach often yields approximate solutions of higher quality than the replacement of  $A$  by  $A_k$ .

## MODAL ANALYSIS IN NON RECIPROCAL WAVEGUIDE BASED ON THE FINITE ELEMENT METHOD

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The study of the electromagnetic fields in photonic components, like optical waveguides, is the key starting point to design novel and advanced optical devices before their manufacturing. For this reason, the development and the optimization of accurate mathematical models is a very important issue.

In this talk, we present an accurate modal analysis for non-reciprocal waveguide using the finite element method. Such a waveguide can be used to perform optical isolator [1], where the forward and backward waves are characterized by a different propagation constants. While the most used method for computing the shift between the forward and the backward propagation constants is the perturbative method [2], here we present a more rigorous approach which allows to directly compute the electromagnetic modes and the corresponding propagation constant [3].

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ASYMPTOTICS OF THE SMALLEST SINGULAR VALUE OF A CLASS OF TOEPLITZ-GENERATED  
MATRICES AND RELATED FINITE RANK PERTURBATIONS

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Square matrices of the form  $X_n = T_n + f_n(T_n^{-1})^*$ , where  $T_n$  is an  $n \times n$  invertible banded Toeplitz matrix and  $f_n$  some positive sequence are considered. The norms of their inverses are described asymptotically as their size  $n$  increases. As an example, for

$$X_n = \begin{bmatrix} 1 + \frac{1}{n} & -1 & 0 & \cdots & \cdots & 0 \\ \frac{1}{n} & 1 + \frac{1}{n} & -1 & 0 & & \vdots \\ \vdots & & & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & -1 & 0 \\ \vdots & \ddots & \ddots & \ddots & & -1 \\ \frac{1}{n} & \cdots & \cdots & \cdots & \frac{1}{n} & 1 + \frac{1}{n} \end{bmatrix},$$

it will be shown that

$$\lim_{n \rightarrow \infty} \frac{2\|X_n^{-1}\|}{\sqrt{n}} = 1.$$

Certain finite rank perturbations of these matrices are shown to have no effect on this behaviour. In the concrete example above, for the matrix  $K_n$  obtained from  $X_n$  by adding one to each entry in the first column, one also has

$$\lim_{n \rightarrow \infty} \frac{2\|K_n^{-1}\|}{\sqrt{n}} = 1.$$

## RATIONAL KRYLOV METHODS AND GAUSS QUADRATURE

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The need to evaluate expressions of the form  $f(A)v$  or  $v^H f(A)v$ , where  $A$  is a large sparse or structured matrix,  $v$  is a vector,  $f$  is a nonlinear function, and  $^H$  denotes transposition and complex conjugation, arises in many applications. Rational Krylov methods can be attractive for computing approximations of such expressions. These methods project the approximation problem onto a rational Krylov subspace of fairly small dimension, and then solve the small approximation problem so obtained. We are interested in the situation when the rational functions that define the rational Krylov subspace have few distinct poles. We discuss the case when  $A$  is Hermitian and an orthogonal basis for the rational Krylov subspace can be generated with short recursion formulas. Rational Gauss quadrature rules for the approximation of  $v^H f(A)v$  will be described. When  $A$  is non-Hermitian, the recursions can be described by a generalized Hessenberg matrix. Applications to pseudospectrum computations are presented.

A SYMBOL APPROACH IN IGA MATRIX ANALYSIS (AND IN THE DESIGN OF EFFICIENT  
MULTIGRID METHODS)

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We study the spectral properties of stiffness matrices that arise when isogeometric analysis is employed for the numerical solution of classical second order elliptic problems. Motivated by the applicative interest in the fast solution (by preconditioned Krylov or multigrid methods) of the related linear systems, we look for a spectral characterization of the involved matrices. In particular, we investigate non-singularity, conditioning (extremal behavior), spectral distribution in the Weyl sense, as well as clustering of the eigenvalues to a certain (compact) subset of the complex field. All the analysis is related to the notion of symbol in the Toeplitz setting and is carried out both for the cases of 1D and 2D problems.

The spectral properties represent the starting point for designing fast two-grid methods for which we provide a numerical confirmation of the optimality, meaning that the spectral radii of the related iteration matrices are bounded by a constant  $c_p$  for all  $n$ ,  $c_p < 1$ : a formal proof of optimality for  $p = 2$  and  $p = 3$  is given. An extension of the results to the two-level case is provided, together with a wide set of numerical tests including the V-cycle and the W-cycle applied to approximated 1D and 2D problems.

## “GOOD” POINTS FOR MULTIVARIATE POLYNOMIAL INTERPOLATION AND APPROXIMATION

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For the interval  $[-1, 1]$ , it is well-known that interpolating in Chebyshev points is much better than using equidistant points in the same interval (Runge phenomenon). This fact forms the basis of Chebfun, a Matlab-toolbox that uses Chebyshev points for the interpolation and Chebyshev polynomials for the representation of the interpolant. Quantitatively, the fact that the Chebyshev points are “good” points for interpolation with a polynomial of degree  $\delta$  corresponds to the fact that the Lebesgue constant grows as  $\log \delta$  while this Lebesgue constant grows much faster in case of equidistant interpolation points. The Lebesgue constant is the maximum of the Lebesgue-function on the geometry considered, in this case the interval  $[-1, 1]$ .

Also for the multivariate case and for different geometries, sets of “good” points were investigated (e.g., Padua points on the unit square) and other “good” point configurations were computed by optimization algorithms. In this talk we will describe an alternative optimization method to compute point configurations with a small Lebesgue constant for different geometries. This method consists of several smaller optimization procedures, taking each more and more computational effort but leading to smaller and smaller Lebesgue constants. It will turn out that the choice of a good basis for a specific geometry is essential to be able to solve the polynomial interpolation problem over that geometry. We will use an orthonormal basis with respect to a discrete inner product where the points of the inner product are lying in the geometry that we are considering at that moment. No explicit representation for these basis polynomials will be computed but we will evaluate them using a recurrence relation, generalizing the three-term recurrence relation on the real line and the Szegő recurrence relation on the complex unit circle.



# A FREE BOUNDARY NUMERICAL METHOD FOR SOLVING AN OVERDETERMINED ELLIPTIC PROBLEM

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This work presents a mathematical problem related to the equilibrium analysis of a prestressed membrane with rigid and cable boundaries. The membrane is represented by a regular surface  $z(x, y)$ , its stresses (tensions) by a positive tensor  $\sigma(x, y)$ , and its boundary by a set of regular curves;  $\Gamma^r$  and  $\Gamma^c$ , i.e. rigid and cable boundary respectively. The membrane-cable interface equilibrium requires taking into account a singular condition on  $\Gamma^c$ , and it makes the problem more difficult. Precisely, if  $H$  represents the Hessian matrix of  $z$  and  $\mathbf{t}$  is the tangent unit vector to  $\Gamma^c$ , once  $\sigma$  is fixed we have to find  $z$  in a bounded domain  $D$  such that

$$\begin{cases} \operatorname{div}(\sigma \cdot \nabla z) = 0 \text{ in } D, \\ z = g \text{ on } \Gamma^r, \quad z = h \text{ on } \Gamma^c \text{ (Dirichlet boundary conditions),} \\ \mathbf{t} \cdot (H \cdot \mathbf{t}) = 0 \text{ on } \Gamma^c \text{ (unusual boundary condition),} \end{cases} \quad (3)$$

$g$  and  $h$  being two functions defined in  $\Gamma^r$  and  $\Gamma^c$  ( $\partial D = \Gamma^r \cup \Gamma^c$ ). In the last system it is not possible to arbitrarily choose both functions  $g$  and  $h$ ; in fact, an overdetermined elliptic problem would be obtained and its solution  $z$  would not necessarily solve also the unusual boundary condition on  $\Gamma^c$ . Therefore, we consider  $\Gamma^c$  as a *free portion* of  $\partial D$  and, by means of an iterative procedure, it is possible to fit the shapes of the cable (i.e.  $\Gamma^c$ ) and of the membrane (i.e.  $z$ ) so that system (3) is completely verified.

The aim of the talk is to define and discuss this mathematical problem and, successively, to present some numerical results regarding the iterative approach.

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## **4. Poster Session**

## PROPAGATION AND INTERACTION OF COHERENT STRUCTURES IN FERROMAGNETIC SYSTEMS

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Magnetic materials yield a rich variety of intriguing nonlinear wave phenomena. Recent theoretical and experimental developments have enabled the controlled manipulation of magnetic moments on the nanometer length scale, the magnetic exchange length, thereby generating further interest in the field of nanomagnetism, particularly as for future spin based information storage and processing technologies. Finally, the generation of coherent and localized magnetic structures (droplet solitons) has been recently experimentally observed, by using spin transfer torque underneath a nanocontact on a magnetic thin film with perpendicular magnetic anisotropy.

The existence, stability, and properties of propagating, (one-droplet) solitary waves in ferromagnetic systems have been inquired into and studied at various times since the first derivation of the corresponding governing equation, the Landau–Lifshitz (LL) equation. Although the LL equation for a one-dimensional uniaxial ferromagnetic system has been shown to be integrable by means of the inverse scattering transform, only the one-droplet solution has been studied extensively in the literature. The research illustrated in this poster is focused on the multi-droplet solutions of the one-dimensional LL equation for an easy-axis ferromagnetic system, in particular on the open problem of describing the propagation and interaction of  $N$ -droplets on the line. The solution to such a problem not only may lead to a better mathematical understanding of the LL equation, but may also provide a deeper physical insight into magnetic phenomena on the nanometer length scale.

## PARAMETER SELECTION STRATEGIES FOR THE ARNOLDI-TIKHONOV METHOD

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In this work we describe some strategies for setting the regularization parameter when performing the Arnoldi-Tikhonov method.

Often, when dealing with linear discrete ill-posed problems of huge dimensions, just a purely iterative or a hybrid approach to regularization can be adopted. The class of the Arnoldi-Tikhonov methods [1] is based on the projection of the original Tikhonov-regularized problem onto Krylov subspaces of small but increasing dimensions; in particular, we are concerned with formulations that can deal with an arbitrary initial guess for the solution and a generic regularization matrix.

In this setting, a suitable value for the Tikhonov regularization parameter should be set at each iteration, as well as a stopping criterion for the underlying Arnoldi algorithm. We present two reformulations of the classical discrepancy principle, including a new scheme that can be applied without any initial estimate on the noise level, which is recovered during the iterations. An efficient reformulation of the Generalized Cross Validation method is presented, too. We briefly address some theoretical estimates, in order to justify our approach [2].

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- [2] S. Gazzola, P. Novati, and M. R. Russo, *Embedded techniques for choosing the parameter in Tikhonov regularization*, submitted (2013).

## CONVERGENCE ACCELERATION OF ROW ACTION METHODS

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During the last decades, the class of the *row projection*, also called *row-action methods* for solving a (possibly large) system of linear equations received much attention since they have several interesting properties (i.e. no changes to the original matrix and no operations on the matrix as a whole). Although the convergence of these methods is quite slow, many acceleration schemes have been proposed, based on different techniques.

Here we present some results already obtained for Kaczmarz's method [2] and new results on Cimmino's method [3]. The acceleration is based on sequence transformations [1]. Two algorithms are proposed: in the first one the accelerated sequence is obtained directly by using the sequence obtained by the original method (Accelerated algorithm); in the second algorithm, the accelerated sequence, is computed by restarting the original method from a vector obtained by an extrapolation method (Restarted algorithm).

Numerical results for both Kaczmarz and Cimmino methods will be presented.

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## AUTOMORPHIC LIE ALGEBRAS WITH DIHEDRAL SYMMETRY

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The idea of *Automorphic Lie Algebras* [1] arose from the concept of reduction groups studied in the early 80s in the field of integrable systems [2]. They are obtained by imposing a discrete group symmetry on a current algebra of Krichever Novikov type. That is, for  $\mathfrak{g}$  a simple Lie algebra,  $\mathcal{M}(\overline{\mathbb{C}})$  the field of meromorphic functions on the Riemann sphere, and  $G$  a finite subgroup of  $\text{Aut}(\mathfrak{g} \otimes \mathcal{M}(\overline{\mathbb{C}}))$ , the Automorphic Lie Algebra is the space of invariants  $(\mathfrak{g} \otimes \mathcal{M}(\overline{\mathbb{C}}))_{\Gamma}^G$  where  $\Gamma \subset \overline{\mathbb{C}}$  is a single  $G$ -orbit where poles are allowed. Past work shows remarkable resemblance between Automorphic Lie Algebras with different reduction groups  $G$  [3], [4]. For example, if  $\mathfrak{g} = \mathfrak{sl}(2, \mathbb{C})$  and  $\Gamma$  is an exceptional orbit,  $|\Gamma| < |G|$ , changing the group does not affect the Lie algebra structure, although the elements of the algebra are different. In the present research we fix  $G$  to be the dihedral group  $\mathbb{D}_N$ , and vary the orbit of poles and the Lie algebra, as well as the  $G$ -action on the Lie algebra. We find a uniform description of these algebras, valid both in the case of generic and exceptional orbits.

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## REGULARIZATION METHODS TO RESOLUTION ENHANCEMENT OF REMOTE SENSED DATA

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Earth-orbiting microwave radiometers are a key remote sensing tool to provide valuable and effective large-scale information for oceanic and atmospheric applications. However, there is a growing interest in other applications that require finer spatial resolution. The resolution of radiometer data can be enhanced by using either image-processing techniques or special reconstruction algorithms. These latter do not enhance the resolution of end-products, as done by ad hoc image-processing procedures; rather, once a low resolution measure of geophysical parameters is provided, they attempt to reconstruct the geophysical parameters on a finer grid. To this aim, a linear ill-posed problem needs to be inverted, which can be physically considered as the analog of an antenna-pattern deconvolution. Hence, regularization methods must be accounted for. In literature several methods to enhance the spatial resolution of radiometer measurements have been proposed, e.g. the Backus-Gilbert, the SIR, the Tikhonov regularization, etc. In this talk, two approaches are proposed:

- A truncated singular value decomposition (TSVD) approach is proposed. The rationale that lies at the basis of the TSVD approach consists of truncating the SVD solution to discard the components dominated by noise. The TSVD is properly extended to the 2D case and shown to be very effective when the kernel is a two-dimensional tensor product.
- An iterative reconstruction technique, based on the gradient method in Banach spaces is proposed. Banach spaces are complete vector spaces endowed with a norm that only allows to measure “length” and “distance” between its elements without any scalar product, that is, without measuring any “angle” between them. The technique is shown to overcome the drawbacks of classical Hilbert space

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## SPECTRAL PROPERTIES OF PERTURBATIONS OF COMPLEX HARMONIC OSCILLATORS

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We investigate the spectral properties of a second order Shubin type differential operator. The main tool is the reduction to global normal forms.

In particular, we are able to describe completely the spectrum of the following model (normal form) non-self-operator on the real line

$$Pu = -u''(x) + (\varepsilon x + p)u' + \omega x^2 + iqx, \quad \varepsilon, p, q \in \mathbb{R}, \omega \in \mathbb{R},$$

which might be viewed as a perturbation of the complex harmonic oscillator studied by E.B. Davies and A.B.J Kuijlaars (2004).

The functional frame of our investigations (in addition to the Schwartz class  $S(\mathbb{R})$  and the weighted Sobolev (Shubin)) is formed by the scale of the Gelfand–Shilov spaces  $S_{\mu, \nu}^{\nu}(\mathbb{R})$ ,  $\mu + \nu \geq 1$ .

*The talk is based on joint work with T. Gramchev (Università di Cagliari).*



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8:30–9:00	REGISTRATION			
Chair:	T. Aktosun	B. Prinari	S. Pennisi	S. Seatzu
9:00–10:00	OPENING	<b>Antonio Degasperis</b> (Plenary) Rational solitons of resonant wave interaction models	<b>Tommaso Ruggeri</b> (Plenary) Recent results in rational extended thermodynamics: macroscopic approach ...	<b>Dario Bini</b> (Plenary) Geometric means of matrices: analysis and algorithms
10:00–10:30	<b>Martin Klaus</b> (Plenary) Spectral problems associated with the matrix-valued AKNS equation	<b>Paolo Santini</b> Integrable multidimensional PDEs of hydrodynamic type: method of solution ...	<b>Giovanni Frosali</b> Nonlinear analysis of the two-mass-skate bicycle model	<b>Lothar Reichel</b> Rational Krylov methods and Gauss quadrature
10:30–11:00		<b>Sara Lombardo</b> Automorphic Lie Algebras	<b>Luigi Barletti</b> Signal-noise interaction in nonlinear optical fibers: a fluid-dynamic approach	<b>Giuseppe Mastroianni</b> Fredholm integral equations on the real semi-axis: a numerical method
11:00–11:20	COFFEE BREAK			
Chair:	G. Biondini	P. Santini	G. Frosali	L. Reichel
11:20–11:50	<b>Folkert Mueller-Hoissen</b> Sylvester equations and integrable systems: a bidifferential calculus perspective	<b>Matteo Sommacal</b> Propagation and control of nanoscale magnetic-droplet solitons	<b>Sebastiano Pennisi</b> Extended Thermodynamics from the Lagrangian view-point	<b>Marc Van Barel</b> "Good" points for multivariate polynomial interpolation and approximation
11:50–12:20	<b>Cornelia Schiebold</b> Multiple-pole solutions of the Nonlinear Schroedinger equation	<b>Marco Secondini</b> Analytical approximations of the NLS equation: applications to ...	<b>Todor Gramtchev</b> Global normal forms and spectral properties for perturbations of harmonic oscillators	<b>Andre Ran</b> Asymptotics of the smallest singular value of a class of Toeplitz-generated matrices ...
12:20–12:50	<b>Annalisa Calini</b> Integrable Flows for Starlike Curves in Centroaffine Space	<b>Luisa Fermo</b> Computation of relevant scattering data in the Zakharov-Shabat system	<b>Salvatore Mignemi</b> Analytical mechanics of a relativistic particle in a positional potential	<b>Michele Benzi</b> Decay properties for functions of matrices over $C^*$ -algebras
12:50–16:00	LUNCH			
Chair:	F. Mueller-Hoissen	A. Degasperis	M. Van Barel	D. Bini
16:00–16:30	<b>Stella Piro-Vernier</b> A Keller-Segel model in chemotaxis with blow-up solutions	<b>Federica Vitale</b> The IST for the defocusing NLS equation with nonzero boundary conditions	<b>Stefano Serra Capizzano</b> A symbol approach in IgA matrix analysis ...	<b>Beatrice Meini</b> On the solution of certain algebraic Riccati equations arising in fluid queues
16:30–17:00	<b>Gino Biondini</b> Scalar and vector nonlinear Schrodinger equations with nonzero boundary conditions	<b>Giovanni Ortenzi</b> Effects of inertia and stratification in incompressible ideal fluids ...	<b>Claudio Estatico</b> Regularized nonconvex minimization for image restoration	<b>Dario Fasino</b> A modularity-based spectral graph analysis
17:00–17:30	<b>Al Osborne</b> Extending integrable methods to nonintegrable evolution equations	<b>Francesco Borghero</b> Some new results in Geometrical Optics	<b>Silvia Noschese</b> A modified TSVD method for discrete ill-posed problems	<b>Caterina Fenu</b> Fast ranking of nodes on digraphs
17:30–17:50	COFFEE BREAK			
Chair:	F. Demontis		G. Rodriguez	
17:50–18:20	<b>Antonio Moro</b> Breaking mechanism from a vacuum point in the defocusing NLS equation	<b>Poster Session</b>	<b>Paolo Pintus</b> Modal analysis in non reciprocal wave-guide based on the finite element method	
18:20–18:50	<b>Georgi Boyadzhiev</b> Characteristic curves in modeling of the earth crust and upper mantle	<b>Visit to the museum of wax anatomical models</b>	<b>Giuseppe Viglialoro</b> A free boundary numerical method for solving an overdetermined elliptic problem	