



# AMALART24

## Advances in Applied Mathematics and Learning Approaches for Cultural Heritage and Arts

Cagliari, Italy  
April 22–24, 2024



Fondazione  
di Sardegna



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The program of the event included plenary and contributed lectures providing both an introductory discussion to applications and cutting edge research results, with the aim of both promoting future collaborations and to stimulate interest in the applied mathematics and AI community.

Archaeology and cultural heritage preservation are revealing themselves as prolific areas of application of novel applied mathematical models and methods. Since the introduction of seriation by the egyptologist W.M.F. Petrie in 1899 for dating archaeological excavation sites and the pioneering work of D.G. Kendall in the late sixties, more and more applications have arisen: computer vision techniques for reconstructing and analyzing bas-reliefs and prehistoric rock art, digital analysis of ancient frescoes and illuminated manuscripts as well as machine and deep learning for the interpretation of ancient documents and archaeological findings, solution of inverse problems in near-surface geophysical survey for detecting buried remains, just to mention a few. A growing interest towards the use of approaches based on applied mathematics, computer vision and learning in the arts rapidly developed over the last decades. This is certainly not limited to visual arts. For example, sound synthesis and filtering in electronic and contemporary music are based on Fourier analysis.

The aim of the symposium is to foster interaction and collaboration among researchers working on applied mathematics and artificial intelligence applications for archaeology and arts. Several research groups in both countries are developing mathematical models, algorithms, and software for problems arising in human sciences, in close interaction with humanities researchers and experts.





## Organizing Committee

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## ANALYSIS-BY-SYNTHESIS FOR HISTORY

**M. Aubry**

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I will present how interpretable analysis-by-synthesis approaches can be leveraged to tackle historical questions. First, I will present a Deep approach to Transformation-Invariant clustering, and illustrate how it can help the analysis of prints, and in particular book ornaments. Second, I will explain how such an approach can be extended to decompose images into multiple visual elements and combined with weak supervision. I will detail how this can be leveraged by paleographers to revisit core historical research questions. I will conclude with what I believe to be an important perspective for the development of analysis-by-synthesis: their potential to complement methods trained on synthetic data, to adapt them to real examples.

## MATHEMATICAL TECHNIQUES FOR FRAGMENT ANALYSIS AND RECOGNITION

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Archaeological excavations return artefacts that often are damaged with parts that are fragmented into more pieces or missing. The comparison of fragments cannot simply be based on the geometric shape, as material, colour, decorations or features, etc. are important factors that concur with this concept [1]. The decorative elements present on an artefact, the thickness of the fragment and the marks left on the object by the tools used to shape it are indicators of the origin and production techniques used and an important element in its cataloguing.

In this talk, we present techniques developed at CNR-IMATI to quantitatively support the analysis, classification and recognition of the fragments found in an archaeological excavation [2, 3]. Our methods range from the identification of geometric features peculiar to a group of objects to the recognition of specific configurations or structures, and the identification and classification of parts with a particular decoration or feature.

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- [2] C. Romanengo, S. Biasotti, B. Falcidieno, *Recognising decorations in archaeological finds through the analysis of characteristic curves on 3D models*, *Pattern Recognition Letters*, 131 (2020), 405–412.
- [3] E. Moscoso Thompson, S. Biasotti, *Description and retrieval of geometric patterns on surface meshes using an edge-based LBP approach*, *Pattern Recognition*, 82 (2018), 1–15.



# ON MATHEMATICAL MODELLING OF NEOLITHIC POTTERY FORMING TECHNIQUES

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Pottery building techniques act as valuable indicators for recognising ancient cultures and practices. The characterisation of the forming sequence allows to retrieve the technical traditions of the past and to identify communities of practice. This presentation will focus on the mathematical characterisation of pottery forming sequence based on tomographic data of archaeological and experimental sherds.

Recent studies [1, 2] will be presented proposing a method for distinguishing between two ancient pottery forming techniques: coiling and Spiralled Patchwork Technology (SPT) [3]. The procedure make use of tomographic 3D data visualisation and processing to examine the internal structure of pottery sherds and to develop a quantitative approach based on the 3D Hough transform. This method could help identify the origins of SPT pottery tradition and track how different traditions spread throughout the Mediterranean and Europe.

## References

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- [2] K. Dia, V.L. Coli, L. Blanc-Féraud, J. Leblond, L. Gomart, D. Binder, *Applications of Learning Methods to Imaging Issues in Archaeology, Regarding Ancient Ceramic Manufacturing*, *Proceedings of the 2nd International Conference on Deep Learning Theory and Applications - DeLTA*, (2021), pp.109–116.
- [3] L. Gomart, D. Binder, L. Blanc-Féraud, L. Cassard, S. Cohen, V.L. Coli, M. Gabriele, J. Leblond, F. Orange, D. Pisani, S. Sorin, *From macrotraces to micro-tomography: a multi-scale approach for detecting and characterizing the "Spiralled Patchwork Technology" in Northern Mediterranean Neolithic pottery assemblages*, *Actes de la Séance de la Société préhistorique française*, 18 (2022).

## ASTRAGALUS BONES IDENTIFICATION VIA TOPOLOGICAL DATA ANALYSIS

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Archaeozoology is a discipline that documents human-animal relationships in the past based on faunal remains found in archaeological contexts. One of the main challenges in this field is to distinguish morphologically close species through archaeological bones. Often, some species belonging to the same family or genus share common morphological features that are difficult to differentiate. We propose a novel approach using topological data analysis (TDA) on the 3D models of astragalus bones of the Caprinae's family. By analyzing the geometric information of the bones, including connected components, cycles and holes, we are able to identify interspecies morphological features that can help distinguish closely related species. We detail the proposed approach and compare it with state of the art methods.

## SOME MATHEMATICAL MODELS FOR UNDERSTANDING AND PREVENTING DAMAGE ON CULTURAL HERITAGE

**R. Natalini**

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In this talk, we present a general survey about a series of works we have done in the last twenty years, with our group, on chemical aggression of stone artifacts. We consider here different problems such as modelling the evolution of gypsum crust in marble stones, the sodium sulphate crystallization inside porous stone (masonry brick), or the effect of the injection of consolidants in stones.

For sulphation and other surface reactions we adapted our previous models to take into account more possible features, as for instance rugosity of the stone and the possible interaction between chemical and mechanical damage, to evaluate the propagation of cracks in stones under stress. For the problem of salt crystallization, a new mathematical model describing the effect of protective products on sodium sulphate crystallization inside bricks has been proposed and tested against experiments. A numerical fitting procedure of the parameters of model for both the case of the brick treated with a protective product and untreated case are implemented. Numerical results show the effectiveness of our method, since we obtain a low average error.

# AI-BASED GENERATION OF A MULTI-MODAL PANORAMA FOR THE BAYEUX TAPESTRY

**Y. Queau**

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The Bayeux Tapestry is an exceptional Middle Age embroidery, of 70m long and 50cm high. Throughout 55 scenes, it tells the epic of William, Duke of Normandy, who left Normandy with his armada in 1066 to conquer the kingdom of England. However, researchers and scientists interested in the study of this unique artifact are confronted with problems related to temporal or geographical constraints i.e., to accessibility: the number of visitors, the exceptional size of the document, the protective glass, etc. In order to solve these accessibility issues and thus facilitate access to the Tapestry to scientists and the general public, we proposed to create a digital multimodal (daylight, multi-spectral and fine-scale geometry) panorama, which can be explored online in a web interface. This talk will present the mathematical and AI tools which were developed for generating this multimodal panorama, from the spatial and spectral registration to the deep learning-based fine-scale 3D-reconstruction.



## A MATHEMATICAL EYE FOR MUSIC THEORY

**M. Sommacal**

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In this talk, intended for a general mathematical audience, after a brief excursus on basic facts of music theory, I will discuss the mathematical representations of harmony (as the branch of music concerned about simultaneously occurring notes and chord sequences) and voice leading (generalising the concept of counterpoint, as the branch of music concerned about sequences of individual notes in a series of chords forming simultaneous melodies) in Western classical music. In particular, I will focus on Euler's Tonnetz, or lattice diagram of the tonal space, and Tymoczko's orbifold theory of musical chords. If time allows, I will briefly touch the problem of automatic classification of musical styles via topological approaches.

## ART MOTIFS IN DOMUS DE JANAS: PURELY STYLISTIC FIGURATIVE EVOLUTION OR EXPRESSION OF CULTURAL CHANGES?

**G. Tanda**

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*This talk will be delivered in French.*

The art of Domus de Janas, funerary hypogea excavated between the 5th and the beginning of 2nd millennium b.C - 7000–4000 years ago - manifests itself with richness and a variety of schematic motifs, executed with different techniques: sculpture, painting and engraving.

Horn-shaped sculpted figures are particularly numerous, in which a hierarchical typology is divided into 3 typological groups (bucranium 1, 2, 3) and 24 types has been recognised. The motifs relating to bucranium 1, in turn, are divided in simple, transitional and complex motifs, among which signs of a figurative evolution have long been detected.

Under this assumption, the simple motifs appear as the closest to the anatomical characteristics of the bovine protome. The transitional motifs mark the stage of the hatch symbolisation, in which the corniform motif, which represents the bovin horns - a part, therefore, of the protome - is merged, on an architectural level, with the hatch.

In this fusion, not only a substitutive figurative function was observed, as the hatch took the place of the bovine's head, but also a conceptual elaboration which is magical-religious and clearly connected with the funerary custom. The hatch, which replaces the head of the bovine animal, has itself become the head of the animal.

Complex motifs are the horn-hatch schemes expanded and projected onto the back wall of the main cell, following the antechamber, occupying it first partially, then entirely, in axial symmetry. The hatch transforms into a false door, constituting the central architectural-cultural element.

Regarding the location of the motifs, we underline their presence prevalent in the pavilion, in the antechamber, in the main cell and, rarely, in a secondary cell, i.e., in environments which have clearly attracted the interest of the artisans and grave excavators, as they are engaging on an ideological-funeral level.

In the antechamber we recognize not only the space that leads to the cell where the deceased is buried, but also a sort of antechamber of the deceased's home, therefore an area of physical separation and, at the same time, of spiritual closeness. For these reasons it is a place where magical-religious rites of propitiation of the souls of the deceased and their divinities are performed, as attested in literature.

The transfer of ritual signs from the antechamber to the next cell marks a cultural change.

The sacred entrance through the divinity and into her world through a real entrance, which takes on the meaning of a symbolic threshold, must have assumed, at a certain moment, the characteristic of a cultic manifestation open to the participation of the community. This resulted

in the search for a larger environment, choosing from those available, that would better respond to the changed needs. Hence the transfer of the ritual ceremonies to the next cell, larger than the antechamber, as a connecting space between cells and disengagement, with the transfer also of the architectural-decorative elements, the "signs" of the cult - including the protomi - restructuring the environment itself.

The archaeological documentation on the hypotheses outlined is neither frequent nor unequivocal. Perhaps interdisciplinary contributions could provide supporting evidence or at least useful clues.

## **References**

- [1] G. Tanda, *Le domus de janas decorate con motivi scolpiti*, Condaghes, Sassari, 2021





## **List of Contributed Talks**



## A VARIATIONAL MODEL FOR INCISIONS AND GLYPHS EXTRACTION

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In archaeology it is a common task to extract incisions or glyphs from a surface. This procedure is usually done manually and, therefore, it is prone to errors and it can be extremely time consuming. In this talk we present a variational model to automatically extract these incisions from a smooth surface.

We model this problem in the following way. Let  $\mathbf{x} \in \mathbb{R}^n$  be a vector containing a sampling of the archaeological surface, we wish to find two vectors  $\mathbf{x}_s^*$  and  $\mathbf{x}_g^*$  such that  $\mathbf{x} = \mathbf{x}_s^* + \mathbf{x}_g^*$ , where  $\mathbf{x}_s^*$  is smooth and contains the background and  $\mathbf{x}_g^*$  is sparse and contains the glyph. To this aim we consider the model

$$\begin{aligned} (\mathbf{x}_s^*, \mathbf{x}_g^*) &= \arg \min_{\mathbf{x}_s, \mathbf{x}_g \in \mathbb{R}^{n \times n}} \frac{1}{2} \|L^\alpha \mathbf{x}_s\|_2^2 + \mu \|\mathbf{x}_g\|_1, \\ &\text{s.t. } \mathbf{x}_s + \mathbf{x}_g = \mathbf{x}, \end{aligned}$$

where  $\mu > 0$ ,  $\alpha \in [1, 2]$ ,  $\|\mathbf{x}\|_p^p = \sum_{i=1}^n |\mathbf{x}_i|^p$ , and  $L \in \mathbb{R}^{n \times n}$  denotes the Laplacian operator. To perform the minimization we employ the ADMM algorithm and we show the performances of the proposed method on synthetic data.

# AN ALTERNATING DIRECTION MULTIPLIER METHOD FOR THE INVERSION OF FDEM DATA

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In this talk, we focus on the numerical solution of nonlinear inverse problems in applied geophysics. Our aim is to reconstruct the structure of the soil, i.e., either its electrical conductivity or the magnetic permeability distribution, by inverting Frequency Domain Electromagnetic (FDEM) data to possibly identify the presence of sites of archaeological relevance. This is a very challenging task since the problem is nonlinear and severely ill-conditioned. To solve the nonlinear inverse problem, we propose an Alternating Direction Multiplier Method (ADMM) algorithm, we prove its convergence, and propose an automated strategy to determine the parameters involved. Moreover, we present two heuristic variations of the ADMM that either improve the accuracy of the computed solutions or lower the computational cost. The effectiveness of the different proposed methods is illustrated through a few numerical examples.

## PHOTOGRAPHIC 3D-SCANNING IN THE WILD

**B. Coupry**, A. Laurent, J. Mélou, Y. Quéau, J.-D. Durou  
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Numerous multi-view 3D reconstruction solutions are available to the general public. However, these methods cannot reconstruct high frequencies as accurately as the so-called photometric stereo (PS) method, which involves estimating the shape and reflectance of a surface from several photographs obtained from the same viewpoint, under different illumination conditions.

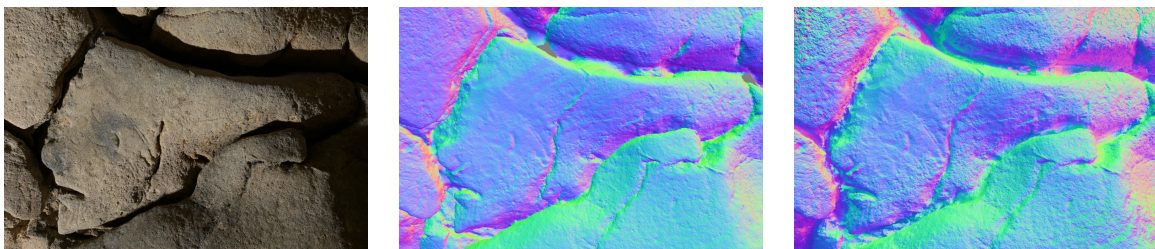


Figure 1: From left to right: One of the 38 images used to reconstruct the archeological artefact "Le masque" (Mas d'Azil, Ariège, France); Normal map estimated by photogrammetry; Normal map estimated by PS using our approach. Fine surface details are more accurately recovered, without losing the overall structure (low frequencies) of the scene.

In order to estimate the illumination conditions at the time of shooting, it is customary to use a sphere. This approach has a number of disadvantages. First, it can be tedious to install a sphere in the scene without obscuring it. Moreover, this method estimates illumination at a single point in the scene. The latter, in the absence of any more reliable information, is then wrongly generalized to the entire scene. Poor illumination estimation degrades the low frequencies of the PS reconstruction. We therefore propose to locally estimate the illumination of the scene from a coarse reconstruction obtained by photogrammetry, in order to improve PS results.

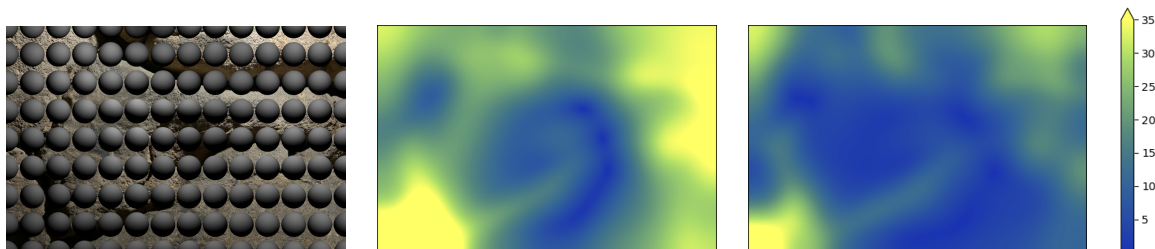


Figure 2: Left: Estimated illuminations at different points of the scene; Center and right: Angular error between PS and photogrammetric reconstructions at low frequencies, respectively using global and local estimations of the illumination.

# A DATA-IDEALITY STUDY FOR PHOTOMETRIC STEREO UNDER UNKNOWN LIGHTING

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Photometric stereo technique allows to reconstruct the three-dimensional shape of an object starting from digital images of it. The method requires that the surface should be Lambertian and the lights should be at infinite distance from the object. In real scenarios this assumptions can not be satisfied, indeed is impossibile to accurately measure the relative position between the light sources and the target. This situation is common in archaeological applications, which is the topic of our studies. Although the Hayakawa method determine an estimation of the light source position starting from the data, in some cases it breaks down because some images deviate from ideality. In order to understand which images from a given dataset should be selected to produce a better reconstruction, we introduce and discuss two measures of data ideality. Then we investigate the performance of these indicators using a synthetic dataset and some experimental datasets.

## NMF-BASED DATA AUGMENTATION FOR GLYPH IMAGES

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The identification and evaluation of inscribed symbols is a challenging problem in archeology and epigraphic studies. In this context, computer vision techniques based on machine learning and data-driven approaches can offer valid solutions to support the work of archeologists. In order to exploit the potential of machine learning strategies based on matrix decomposition for the task of glyphs clustering and classification, huge volumes of data are needed. To this aim, in this work we investigate the application of affine transformations on non-negative factorizations (see e.g. [1]) of the image to provide simultaneously a new image and its factorization, in a data augmentation fashion (see e.g. [2]).

### References

- [1] Z. Y. Zhang, *Nonnegative matrix factorization: models, algorithms and applications*, Data Mining: Foundations and Intelligent Paradigms: Volume 2: Statistical, Bayesian, Time Series and other Theoretical Aspects, (2012), pp. 99–134.
- [2] C. Shorten, T. M. Khoshgoftaar, *A survey on image data augmentation for deep learning*, Journal of Big Data, 6(60) (2019), pp. 1–48.

## AN APPLICATION OF THE SERIATION PROBLEM TO THE DOMUS DE JANAS

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Seriation is an ordering problem which consists in determine the best ordering of a set of correlated units, whose relationship is defined by a bipartite graph. It has several applications in different fields, like genetics, antropology, archaeology and psychology. Our application consists in ordering archaeological sites based on the characteristics of parietal engravings found in them, using the Matlab package in [1]. During the application we considered two matrices, which gave us information about the similarity among the data. We observed that the method (due to its formulation) in some cases extract a solution in which some data seem not be in the correct position. So we determined an indicator to select the best order among different ones and apply it to order the *Domus the Janas* sites, for determining a chronological sequence of them.

### References

- [1] A. Concas, C. Fenu, G. Rodriguez, *PQser: a Matlab package for spectral seriation*, Numerical Algorithms, 80 (2019), pp. 879–902.



## EFFICIENT SOLUTION OF A METAL CORROSION MODEL

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Preventing corrosion damages is paramount for the conservation of historical metal artefacts. The development of mathematical tools and computational experiments can enhance the comprehension and control of corrosive phenomena, which is crucial for devising new techniques for material protection and predictive maintenance.

Metal surfaces are usually covered by a protective film. However, this film may locally break, causing an exposure of the metal to the environment that may result in a localized corrosion attack known as pitting corrosion.

In this talk we consider a phase field model for pitting corrosion [3]. Phase field models allow for a simple treatment of moving interfaces, but the equations to be solved are highly stiff and their efficient solution is challenging.

We present a new numerical approach for the solution of this problem, based on an efficient time integrator for the stiff system of ODEs obtained from a finite difference space discretization. We consider some benchmark problems to compare the efficiency and accuracy of the new approach with other techniques from the literature [1, 2].

### References

- [1] D. Conte, G. Frasca-Caccia, *A Matlab code for the computational solution of a phase field model for pitting corrosion.*, Dolomites Research Notes on Approximation, 15.2 (2022), pp. 47–65.
- [2] H. Gao, L. Ju, R. Duddu, H. Li, *An efficient second-order linear scheme for the phase field model of corrosive dissolution*, Journal of Computational and Applied Mathematics, 367 (2020), 112472
- [3] W. Mai, S. Soghrati, R. G. Buchheit, *A phase field model for simulating the pitting corrosion*, Corrosion Science, 110 (2016), pp. 157–166.

# FROM PIXEL TO PATTERN: TRANSFORMING BELL-BEAKER POTTERY ANALYSIS WITH AI

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This work weaves a narrative where advanced technology meets ancient artifacts, shedding light on the intricate Bell-Beaker pottery that has mystified historians and archaeologists alike. Bell-Beaker pottery, known for its distinctive bell-shaped profiles and richly decorated surfaces, serves as a testament to the sophisticated craftsmanship and cultural interconnectedness of prehistoric communities. The intricate patterns and motifs adorning these vessels have long been thought to hold keys to understanding the social structures, trade networks, and technological advancements of the Beaker culture, which spread across Western Europe during the late Neolithic to early Bronze Age.

Leveraging the prowess of artificial intelligence, this study harnesses the power of computer vision and unsupervised machine learning to unravel the complexity of Bell-Beaker decorations. Through a meticulous orchestration of image processing functions and clustering algorithms, the research aims to classify these ancient artifacts based on the intricacy and designs of their decorations.

At the heart of this exploration lies a suite of specialized image processing functions. Computer vision techniques are employed to analyze the spacing between bands, detect zigzag patterns and complexity and intersections quantification. These processes are instrumental in extracting meaningful patterns and features from the decorations, offering glimpses into the cultural, chronological, or functional contexts of the pottery. The feature extraction phase meticulously gathers texture properties, edge characteristics, and other pertinent features from the images. Subsequent clustering and analysis employ methods, grouping images based on extracted features and visualizing them effectively with PCA and SOM.

The study delves into anomaly detection and evaluates clustering effectiveness through some metrics to gather data about the performance. The insights gleaned from these analyses illuminate new pathways in the categorization of Bell-Beaker pottery. This research not only highlights the integration of artificial intelligence with archaeological investigations but also underscores the invaluable contributions of unsupervised learning and computer vision to the preservation of cultural heritage. “From Pixel to Pattern” is a proposal that to the merges past and present, where ancient artifacts and modern technology unite to uncover the enduring legacies of human communities. Through this lens, the Bell-Beaker pottery of the Iberian Peninsula is reinterpreted, offering a fresh perspective.

## MUSICBO, A MUSICAL HERITAGE KNOWLEDGE GRAPH

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The MusicBO Knowledge Graph (KG), documenting Bologna’s pivotal role in the European Musical Heritage, is created using an automated text-to-Knowledge Graph pipeline on a multilingual and diachronic corpus. The corpus contains 137 documents in English, French, Italian, and Spanish. The documents’ publication dates span from 1700 to the current era. This corpus includes an extensive variety of textual genres, such as historical dissertations, critical analyses, correspondences, and business-oriented documents. The KG has been created automatically by leveraging text2AMR2FRED<sup>1</sup> [2], a revised architecture of FRED’s text-to-KG pipeline [3]. For creating MusicBO KG, we applied the mentioned pipeline to a subset of the mentioned corpus, made of 47 documents in English and 51 documents in Italian. The documents were originally in .pdf, images, or .docx formats. We extrapolated plain text from them through ad hoc Optical Character Recognition (OCR) technologies. We then performed co-reference resolution, rule-based minimal post-OCR corrections, and sentence splitting on the extrapolated plain texts. We submitted the pre-processed sentences to State-of-the-Art (SotA) neural models for text2AMR parsing. For sentences in English, we used SPRING<sup>2</sup> [1]. For sentences in Italian, we used USEA<sup>3</sup> [5]. AMR graphs, grounded in PropBank Frames<sup>4</sup>, serve as an intermediate event-centric representation, well-suited for retrieving ‘who-did-what-to-whom’ in a text. The produced AMR graphs are filtered based on automatic metrics. The remaining AMR graphs undergo AMR-to-FRED translation, which exploits the AMR2FRED tool<sup>5</sup> [4] to transform AMR graphs into OWL-compliant RDF KGs, following FRED’s theoretical model<sup>6</sup>. The resulting KG provides scholars, particularly musicologists, with access to structured information extracted from historical and multilingual sources, poised to potentially enhance their research and understanding of the musical heritage domain. In particular, statistics about triples and data stories can be extrapolated from the KG. Some examples are provided. The proposed KG is freely accessible at <https://polifonia.disi.unibo.it/musicbopilot/query>.

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<sup>1</sup><https://arco.istc.cnr.it/txt-amr-fred/>

<sup>2</sup><https://nlp.uniroma1.it/spring/>

<sup>3</sup><https://github.com/SapienzaNLP/usea>

<sup>4</sup><http://propbank.github.io/v3.4.0/frames/>

<sup>5</sup><http://framester.istc.cnr.it/amr-2-fred>

<sup>6</sup><http://wit.istc.cnr.it/stlab-tools/fred/demo/>

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# GENERALIZING TO UNSEEN PAINTINGS WHEN CLASSIFYING 14TH-CENTURY PUNCH MARKS

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Punch marks are decorative impressions created by hammering a tool with a pattern on one end into gold ground paintings. Intricate punch marks started becoming widespread during the 14<sup>th</sup> and 15<sup>th</sup> centuries in Italy. The tools to create punch marks were often used in multiple paintings and were sometimes shared with other workshops, making punch mark classification useful in various art-historical investigations. While standard convolutional neural networks have shown the capacity to excel at automatically classifying punch marks, they can become substantially less accurate when tested on images of punch marks from panels unseen in the training data [1].

Framing the problem as one of domain generalization, we evaluate several methods on their ability to classify punch marks from panels not in the training data. We use images from [1] alongside photographs and microscopy images from 14<sup>th</sup>-century panels by Simone Martini currently at the Fitzwilliam Museum, University of Cambridge, and at the Wallraf-Rischartz Museum in Cologne, Germany. In this context, we find that domain generalization methods can yield remarkable improvements in accuracy for unseen panels, potentially achieving accuracies above 70% where other standard convolutional neural networks do not exceed 40%. These results illustrate the importance of domain generalization in this setting and suggest the potential to aid art historians and conservators in classifying punch marks.

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# REGULARIZED LIN ELECTROMAGNETIC DATA INVERSION VIA A 2D FIRST-KIND INTEGRAL MODEL

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The aim of this work is to reconstruct the electrical conductivity of the subsoil by Frequency Domain Electromagnetic (FDEM) data inversion. To this end, a 2D first-kind integral model which describes the interaction between the soil and an electromagnetic device (usually known as Ground Conductivity Meter) is derived.

On one hand, the resolution of the forward problem is based on the Gauss-Legendre quadrature formula obtaining accurate approximations of the apparent conductivity of the soil. On the other hand, to solve the inverse problem, we discretize the integrals involved in the model leading to a severely ill-conditioned linear system. To overcome this difficulty, we regularize by using a Tikhonov method and we apply different ways to choose the regularization parameter.

Different numerical experiments on synthetic data are presented to show the effectiveness of the entire approach.

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# SURFACE RECONSTRUCTION FROM POINT CLOUD USING A SEMI-LAGRANGIAN SCHEME WITH LOCAL INTERPOLATOR

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We propose a level set method to reconstruct unknown surfaces from point clouds, without assuming that the connections between points are known. This issue is particularly relevant in the field of cultural heritage where it is common to have a set of unorganized points obtained via laser scanning techniques as input data to model the shape of a work of art, both for visualization purposes and for further investigations. From a mathematical point of view, the problem follows a variational and partial differential equation (PDE) formulation with a curvature constraint that minimizes the surface area weighted by the distance of the surface from the point cloud. Level set methods are used in this framework to track the evolution of an initial surface and to find an implicit representation of the final shape. Among all the possible representations, we compute the signed distance function at least in the vicinity of the reconstructed surface. The numerical method for the approximation of the solution is based on a semi-Lagrangian scheme whose main novelty is its coupling with a local interpolator, with the aim of saving computational costs, considering in particular a multi-linear interpolator and a WENO one to improve the accuracy of the reconstruction. Special attention has been paid to the localization of the method and to the development of fast algorithms that run in parallel, resulting in faster reconstruction and thus the opportunity to easily improve the resolution. A preprocessing of the point cloud data is also performed to set the parameters of the method in a suitable way. Numerical tests in two and three dimensions are presented to evaluate the quality of the approximated solution and the efficiency of the algorithm in terms of CPU times.

# 3D INPAINTING VIA OSMOTIC FLOW OF DIFFERENTIAL COORDINATES

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Digitalized archaeological finds and fragments are used for preservation, accessibility, enhanced analysis, visual representation as well as blueprints for restoration efforts. Elaborating fragments digitally, archaeologists can, for example, virtually piece together the artifact's complete form.

The initial artifact reconstruction presumably contains multiple types of damage, apart from the fractures between individual fragments. Typically, inpainting is the editing tool used for correction of corrupted regions of the mesh.

This work proposes a novel approach to 3D mesh inpainting, which combines the deformation capacity of the PDE Osmotic-flow [1], with the shape representational power of the differential coordinates [2]. The osmotic-flow applied to intrinsic shape descriptors allows to smoothly recover the internal damaged region, producing healed shape reconstruction.

The preliminary result offer a promising direction for various archaeological, scientific and engineering applications.

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Monday, April 22		Tuesday, April 23		Wednesday, April 24	
9:00-9:20	Opening	M. Cornelli	R. Natalini		
9:20-9:50	G. Tanda	Y. Queau	S. Biasotti		
9:50-10:10	C. Fenu	Coffee Break	Coffee Break		
10:10-10:40	Coffee Break	B. Coupry	G. Frasca-Caccia		
10:40-11:10	W. E. Peaslee	E. Crabu	J. Jiménez-Puerto		
11:10-11:40	M. Aubry	S. Crisci	G. Recupero		
11:40-12:10					
12:10-12:40	Lunch	Lunch	Lunch		
12:40-15:00					
15:00-15:30	V. L. Coli		S. Preda		
15:30-16:00			A. Azzarelli		
16:00-16:30	Coffee Break	Excursion	Coffee Break		
16:30-17:00	M. Sommacal		A. Buccini		
17:00-17:30			F. Pes		
17:30-18:00	A. Meloni				
18:00-21:00					