Variational and Krylov methods for the solution of discrete inverse problems

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The discretization of Fredholm integral equation of the first kind often leads to linear systems of equation of the form

$$A\mathbf{x} + \boldsymbol{\eta} = \mathbf{b}^{\delta},$$

where $A \in \mathbb{R}^{m\times n}$ is a severely ill-conditioned matrix, i.e., such that its singular values rapidly decrease to zero with no significant gap between the smallest ones, $\boldsymbol{\eta} \in \mathbb{R}^m$ collects measurement and discretization errors and is often referred to as *noise*, $\mathbf{b}^{\delta} \in \mathbb{R}^m$ represents the measured data, and $\mathbf{x} \in \mathbb{R}^n$ is the unknown quantity we wish to recover. Problems of this form are referred to as *discrete inverse problems*. The naive solution of these is usually meaningless and one needs to regularize the problem. Regularization methods substitute the original problem with a well-posed one that is less sensitive to the perturbations in the data and whose solution is a good approximation of the desired one.

In this lecture we will present efficient and accurate algorithms for the approximate solution of discrete inverse problems. The regularized version will be formulated as a variational problem and we will propose model reduction strategies, based on the projection into Krylov and Generalized Krylov subspaces, to efficiently solve them. Moreover, we will show theoretical results to justify the chosen model and algorithms.