Product Integration Rules on Plans Domains

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Abstract

This talk deals with the numerical computation of integrals of the type

$$I(f;\mathbf{t}) = \int_D f(\mathbf{x})K(\mathbf{x},\mathbf{t})w(\mathbf{x})d\mathbf{x}, \qquad \mathbf{x} = (x_1,x_2), \quad \mathbf{t} = (t_1,t_2), \tag{1}$$

where $D = [-1,1] \times [-1,1]$, f is a sufficiently smooth bivariate function defined on D, with possible algebraic singularities on the boundaries of D, w is the product of two Jacobi weight functions and the kernel K is defined in $D \times T$, where $T \subseteq D$. Kernels of this type, appear, for instance, in the numerical solution of integral for BEM 3D (see [1], [4]) and in the numerical solution of bivariate integral equations (see [2], [3]). We study the product cubature rule obtained by approximating the "regular"function f by suitable bivariate Lagrange polynomials interpolating the smooth function f at the zeros of univariate Jacobi polynomials. For some choices of the kernel K we determine suitable strategies to compute the coefficients of the cubature rule. For instance, we treat the cases of "nearly"singular kernels of the type

$$K(\mathbf{x},c) = \frac{1}{(x_1^2 + x_2^2 + c^2)^\lambda}, \quad 0 < |c|, \quad \lambda > 0,$$

for "small"values of the parameter c and of kernels like

$$K(\mathbf{x},\omega) = g(\omega x_1 x_1),$$

where g is a smooth high oscillatory function for "large"values of the parameter ω.

The stability and the convergence of the cubature rule are proved and some numerical tests, which confirm the theoretical estimates, are proposed.

Acknowledgements. This research was supported by the *University of Basilicata*, by the *Centro Universitario Cattolico* (CUC) and by the *GNCS*.

References

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