# A STRUCTURE-PRESERVING UPWIND DG SCHEME FOR A DEGENERATE PHASE-FIELD TUMOR MODEL

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The purpose of this talk is to present an upwind DG approximation of a degenerate phase-field tumor model that preserves the physical properties of the continuous model.

The tumor model that we consider is based on the one presented in [3], which consists of a Cahn-Hilliard equation for the tumor variable and a diffusion equation for the nutrient variable, coupled by proliferation and cross-diffusion terms. In this sense, we propose several modifications to this model that impose important physically meaningful bounds on the phase-field variable.

Then, we extend the work in [2, 1] to develop a nonlinear discrete scheme based on a convex-splitting time discretization and an upwind discontinuous Galerkin (DG) spatial scheme that approximates the solution of the continuous model preserving its physical properties (mass conservation, pointwise bounds and energy stability).

Finally, we use FEniCSx and its built-in Newton's method to approximate the nonlinear scheme and carry out several numerical experiments. These tests compare our DG scheme with a continuous finite element spatial discretization, which presents numerical spurious oscillations due to the cross-diffusion terms. In addition, they show the behavior of the model under different choices of parameters, mobility and proliferation functions.

## References

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[3] A. Hawkins-Daarud, K. G. van der Zee, and J. T. Oden, *Numerical simulation of a thermodynamically consistent four-species tumor growth model*, International Journal for Numerical Methods in Biomedical Engineering, 28 (2012), pp. 3–24.