

A SMOOTHING ANALYSIS FOR MULTIGRID METHODS APPLIED TO TEMPERED FRACTIONAL PROBLEMS

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We consider the numerical solution of time-dependent space-tempered fractional diffusion equations. The use of Crank-Nicolson in time and of second-order accurate tempered weighted and shifted Grünwald difference in space leads to dense (multilevel) Toeplitz-like linear systems. By exploiting the related structure, we design an ad-hoc multigrid solver and multigrid-based preconditioners, all with weighted Jacobi as smoother. A new smoothing analysis is provided, which refines state-of-the-art results expanding the set of suitable Jacobi weights. Furthermore, we prove that if a multigrid method is effective in the non-tempered case, then the same multigrid method is effective also in the tempered one. The numerical results confirm the theoretical analysis, showing that the resulting multigrid-based solvers are computationally effective for tempered fractional diffusion equations.