

# ON THE ROAD... TO THE TREATMENT OF NEAR-BREAKDOWN IN THE CONJUGATE GRADIENT ALGORITHM

**C. Brezinski** and M. Redivo-Zaglia  
Laboratoire Paul Painlevé, University of Lille  
France  
`claude.brezinski@univ-lille.fr`

The conjugate gradient algorithm for solving systems of linear equations with a symmetric positive definite matrix was obtained in 1952 by Magnus Rudolph Hestenes (1906-1991) and Eduard Stiefel (1909-1978).

It is a direct method since, for a system of dimension  $N$ , it converges in  $N$  iterations at most. When  $N$  is large, it is used as an iterative method.

The conjugate gradient algorithm makes use of two recurrence relations each of them depending on a rational coefficient changing at each step. It cannot suffer from a breakdown due to a division by zero in the computation of the coefficients of the recurrence relations.

However, the algorithm can suffer from a near-breakdown when the denominator of one of these coefficients (or when the coefficient itself) is close to zero, thus leading to rounding errors and non-convergence.

First, we will give some academic examples showing that a near-breakdown really arises. Then, we will derive the recurrence relations for jumping over it.

This work is unfinished since we have not yet programmed it, and we have no numerical results to present.