A VARIATIONAL NON-LINEAR CONSTRAINED MODEL FOR THE INVERSION OF FDEM DATA

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In this talk we consider the reconstruction of the electrical conductivity of the ground using Frequency Domain Electromagnetic (FDEM) induction devices. This nonlinear ill-posed inverse problem is of the form

$$\boldsymbol{\Sigma} = \arg\min_{\boldsymbol{\Sigma}} \sum_{j=1}^{N} \left\| M(\boldsymbol{\sigma}_j) - \boldsymbol{b}_j \right\|_2^2, \tag{1}$$

where $\boldsymbol{\Sigma} = [\boldsymbol{\sigma}_1, \dots, \boldsymbol{\sigma}_N]$ collects the electrical conductivity at certain depths, $\boldsymbol{B} = [\boldsymbol{b}_1, \dots, \boldsymbol{b}_N]$ are the measured data, and the vector function $\boldsymbol{M}(\boldsymbol{\Sigma})$ returns the readings predicted by the model.

Even though the model is separable, treating each $\boldsymbol{\sigma}_j$ separately creates artifacts in the reconstructions. To remove them we consider the following variational problem

$$\arg\min_{\boldsymbol{\Sigma} \geq 0} \frac{1}{2} \left\| \boldsymbol{M}(\boldsymbol{\Sigma}) - \boldsymbol{B} \right\|_F^2 + \frac{\gamma}{q} \left\| \boldsymbol{D}(\boldsymbol{\Sigma}) \right\|_q^q, \tag{2}$$

where $\gamma > 0$, $0 < q < 1$, and $\boldsymbol{D}(\boldsymbol{\Sigma})$ is the two-dimensional Laplacian of $\boldsymbol{\Sigma}$.

In this talk we present the results obtained in [1]. Firstly, we describe a new variational model for (1). Secondly, we prove the regularization properties of $\ell^2 - \ell^q$ in the nonlinear case. Finally, we show the advantages of the proposed approach on both synthetic and real data.

References

[1] A. Buccini and P. Díaz de Alba. *A variational non-linear constrained model for the inversion of FDEM data*. Inverse Problems 38 (2022), p. 014001.