A NEW FAST NYSTRÖM METHOD FOR SOLVING LINEAR VOLTERRA INTEGRAL EQUATIONS ON INFINITE INTERVALS

I. M. Bulai, **M. C. De Bonis**, and C. Laurita
Department of Mathematics, Computer Sciences and Economics,
University of Basilicata
Viale dell'Ateneo Lucano 10, 85100 Potenza, Italy
mariacarmela.debonis@unibas.it

Recently in [1, 2, 3] a numerical method has been introduced for the numerical solution of the following linear Volterra integral equations of the second kind on the half line

$$f(t) - \int_0^t k(s,t) f(s)\, ds = g(t), \quad t \geq 0,$$

where the kernel $k(s,t)$ is a given function defined on $\Delta = \{(s,t) \,|\, 0 \leq s \leq t\}$, $g(t)$ is a known function on $\mathbb{R}^+$ and $f(t)$ is the unknown solution. This method is of Nyström type and consists in discretizing the integral operator by a product quadrature rule based on a truncated Lagrange interpolation process. The involved modified moments, depending on the kernel function, are approximated by a truncated gaussian quadrature formula.

The aim of this talk is to propose a new efficient Nyström method that is much faster than the previous one. It is also based on a product type quadrature rule, but it has the advantage of requiring the computation of modified moments that are independent of the kernel function and can be computed exactly a priori. This produces a great saving of the computational cost.

Such kind of integral equations are of interest because they play an important role in many areas of sciences and, in particular, arise from the reformulation of differential models describing metastatic tumor growth whose unknown solutions represent biological observables as the metastatic mass or the number of metastases [4].

We give sufficient conditions under which the method is convergent in suitable weighted spaces of continuous functions and lead to solve well conditioned linear systems. Numerical examples showing its reliability as well as comparisons with the results obtained with the previous method are presented.

References

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