ITERATIVE REGULARIZATION IN BANACH SPACES

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It is well-known that one-step iterative gradient methods to minimize the residual functional $\Phi(x) = \frac{1}{p}\|Ax - y\|_Y^p$, with $p > 1$, act as implicit regularization algorithms for solving the functional equation $Ax = y$ characterized by an ill-posed linear operator $A : X \longrightarrow Y$ between two Banach spaces X and Y, when combined with an early-stopping criterion to prevent over-fitting of the noise on the data y.

We show that gradient-based minimization of the residual, involving both the dual spaces X^* and Y^*, can be fully understood in the context of proximal operator theory, with suitable Bregman distances as proximity measure [1, 3]. Moreover, some relationships with classical projection algorithms of numerical linear algebra, such us Cimmino and Kaczmarz ones, are discussed [2]. After reviewing the key concepts of modular and duality map, we apply Banach spaces iterative regularization to deblurring and subsurface prospecting ima-ging problems.

References

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