A DEFECT-CORRECTION ALGORITHM FOR QUADRATIC MATRIX EQUATIONS, WITH APPLICATIONS TO QUASI-TOEPLITZ MATRICES

D.A. Bini and **B. Meini**
Department of Mathematics, University of Pisa, Italy
beatrice.meini@unipi.it

A defect correction formula for the quadratic matrix equation $A_1X^2 + A_0X + A_{-1} = 0$ is presented. More specifically, assume that $\widetilde{G}$ is an approximation of the sought solution G. Then, by following the ideas of [2] and [3], we derive an equation for the defect $H = G - \widetilde{G}$ and express H in terms of an invariant subspace of a suitable pencil. This equation allows us to introduce a modification of the Structure-preserving Doubling Algorithm (SDA), that enables refining an initial approximation to the sought solution.

Finally, we show an application to the analysis of random walks in the quarter plane, where the matrix coefficients A_i, $i = -1, 0, 1$, as well as the sought solution G, are infinite matrices endowed of the quasi-Toeplitz structure (QT matrices). Numerical experiments confirm the effectiveness of the proposed method. More details can be found in [1].

References

[1] D. A. Bini, B. Meini, *A defect-correction algorithm for quadratic matrix equations, with applications to quasi-Toeplitz matrices*, 2022; arXiv:2212.09491.

[2] A. Bunse-Gerstner, V. Mehrmann, *A symplectic QR like algorithm for the solution of the real algebraic Riccati equation*. IEEE Trans Automat Control. (1986) 31(12):1104–1113.

[3] V. Mehrmann, E. Tan, *Defect correction methods for the solution of algebraic Riccati equations*. IEEE Trans Automat Control. (1988) 33(7):695–698.