SIMULTANEOUS APPROXIMATION OF HILBERT AND HADAMARD TRANSFORMS ON BOUNDED INTERVALS

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In this talk we propose a method to approximate the weighted Hilbert and Hadamard transforms of a given function f

$$\mathcal{H}_0(fw,t) = \fint_{-1}^{1} \frac{f(x)}{x-t} w(x)dx \qquad \mathcal{H}_1(fw,t) = \fint_{-1}^{1} \frac{f(x)}{(x-t)^2} w(x)dx \tag{1}$$

where $t \in (-1,1)$ and $w(x) = (1-x)^\alpha(1+x)^\beta$, $\alpha,\beta > -1$ is a Jacobi weight.
The proposed method is obtained by means of a simultaneous approximation technique involving the extended Lagrange interpolation process (cf. [1]) based on the zeros of the polynomial $p_{m+1}(w)p_m(w)$, where $\{p_n(w)\}_n$ denotes the orthonormal sequence w.r.t the weight w. Moreover, we introduce a scheme that combines the product formula based on the zeros of $p_m(w)$ and the above introduced extended product rule. Such scheme allows a significant reduction in the number of evaluations of the function f. The numerical stability and convergence of this combined scheme are proved in suitable weighted uniform spaces. Finally, some numerical tests are presented in order to highlight the efficiency of the combined scheme and to confirm the theoretical estimates.

References

[1] G. Criscuolo, G. Mastroianni, D. Occorsio. *Convergence of Extended Lagrange Interpolation*, Mathematics of Computation, 55(191) (1990), pp. 197–212.