NETWORK ANALYSIS WITH THE AID OF THE PATH LENGTH MATRIX

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Let a network be represented by a simple graph $\mathcal{G}$ with n vertices. A common approach to investigate properties of a network is to use the adjacency matrix $A = [a_{ij}]_{i,j=1}^{n} \in \mathbb{R}^{n\times n}$ associated with the graph $\mathcal{G}$, where $a_{ij} > 0$ if there is an edge pointing from vertex v_i to vertex v_j, and $a_{ij} = 0$ otherwise. Both A and its positive integer powers reveal important properties of the graph. This talk proposes to study properties of a graph $\mathcal{G}$ by also using the path length matrix for $\mathcal{G}$. The $(ij)^{th}$ entry of the path length matrix is the length of the shortest path from vertex v_i to vertex v_j; if there is no path between these vertices, then the value of the entry is ∞. Powers of the path length matrix are formed by using min-plus matrix multiplication and are important for exhibiting properties of $\mathcal{G}$. We show how several known measures of communication such as closeness centrality, harmonic centrality, and eccentricity are related to the path length matrix, and we introduce new measures of communication, such as the harmonic K-centrality and global K-efficiency, where only (short) paths made up of at most K edges are taken into account. The sensitivity of the global K-efficiency to changes of the entries of the adjacency matrix also is considered.