Review of the convergence of the conjugate gradient method

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Krylov subspace methods [1, 2, 3] are widely used to iteratively solve a variety of linear systems of equations with one or several right-hand sides, or for solving nonsymmetric eigenvalue problems.

The Conjugate Gradient (CG) method is known as one of the best iterative methods for solving symmetric positive definite linear systems. It generates a symmetric triangular matrix with a specific structure that can be very helpful in understanding the convergence behavior of the conjugate gradient method. Its study also provides an interesting alternative to Chebyshev polynomials.

We will also provide a sample formula based on a rational function for the A-norms of the error and residual norms, based on the eigenvalue decomposition of the matrix and the right-hand side. By minimizing this rational function over a convex subset, we can obtain sharp bounds. We use techniques from constrained optimization rather than solving the classical min-max problem.

References

[1] J. Liesen, Z. Strakos, *Krylov subspace methods: principles and analysis*, Numer. Math. and Scientific Comput., Oxford University Press, first edition (2013).

[2] G. Meurant and J. Tabbens, *Krylov methods for non-symmetric linear systems: From theory to computations,* Springer Series in Computational Mathematics, 57 (2020).

[3] Y. Saad, *Iterative Methods for Sparse Linear Systems*. The PWS Publishing Company, Boston, 1996. second edition, SIAM, Philadelphia, (2003).