Computationally Efficient Methods for Large-Scale Inverse Problems: From Learning to Sparsity and Edge-Preserving

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Abstract: Inverse problems are ubiquitous in many fields of science such as engineering, biology, medical imaging, atmospheric science, and geophysics. Three emerging challenges on obtaining relevant solutions to large-scale and data-intensive inverse problems are ill-posedness of the problem, large dimensionality of the parameters, and the complexity of the model constraints [1, 3]. In this short course we take a journey by introducing large-scale inverse problems and applications, discuss potential challenges, and recent contributions on deterministic and learning methods. In particular, in the fist part of the course we discuss efficient methods for computing solutions to dynamic inverse problems [4], where both the quantities of interest and the forward operator may change at different time instances. We consider large-scale ill-posed problems that are made more challenging by their dynamic nature and, possibly, by the limited amount of available data per measurement step. To remedy these difficulties, we apply efficient regularization methods that enforce simultaneous regularization in space and time (such as edge enhancement at each time instant and proximity at consecutive time instants) and achieve this with low computational cost and enhanced accuracy. In the remainder of the course, we focus on computationally efficient methods that learn optimal ℓ_p and ℓ_q norms for $\ell_p - \ell_q$ regularization and learn optimal parameters for regularization matrices defined by covariance kernels [2]. Numerical examples from a wide range of applications, such as tomographic reconstruction and image deblurring are used to illustrate the effectiveness of the described approaches. Python and Matlab examples will be used to illustrate the topics covered in the course.

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ShortBio Dr. Pasha is a National Science Postdoctoral Fellow at the Department of Mathematics at Tufts University. She develops algorithms and numerical methods for large scale inverse problems. Her research is strongly focused on numerical linear algebra, but she also use techniques and tools from statistics, numerical optimization, and partial differential equations. Furthermore, outside of inverse problems, she is using her computational skills to advance research in areas such as tensor decompositions and computational statistics. Dr. Pasha obtained her Ph.D. from Kent State University in 2020. After graduation, she was a postdoc at the School of Mathematical Sciences at Arizona State University until May 2022.

References

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